Chapter 9A

Process Capability & SPC

OBJECTIVES

- Process Variation
- Process Capability
- Process Control Procedures
  - Variable data
  - Attribute data
- Acceptance Sampling
  - Operating Characteristic Curve

Basic Forms of Variation

Assignable variation is caused by factors that can be clearly identified and possibly managed

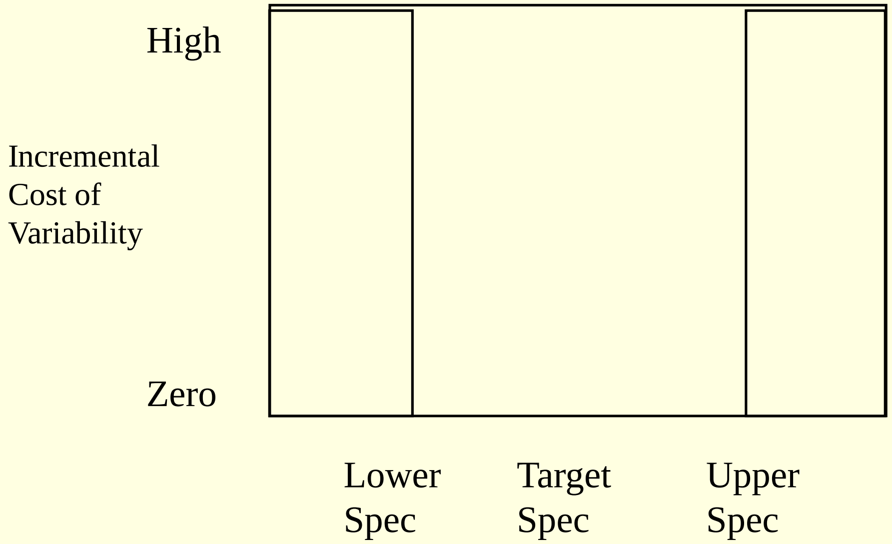
Example: A poorly trained employee that creates variation in finished product output.

Common variation is inherent in the production process

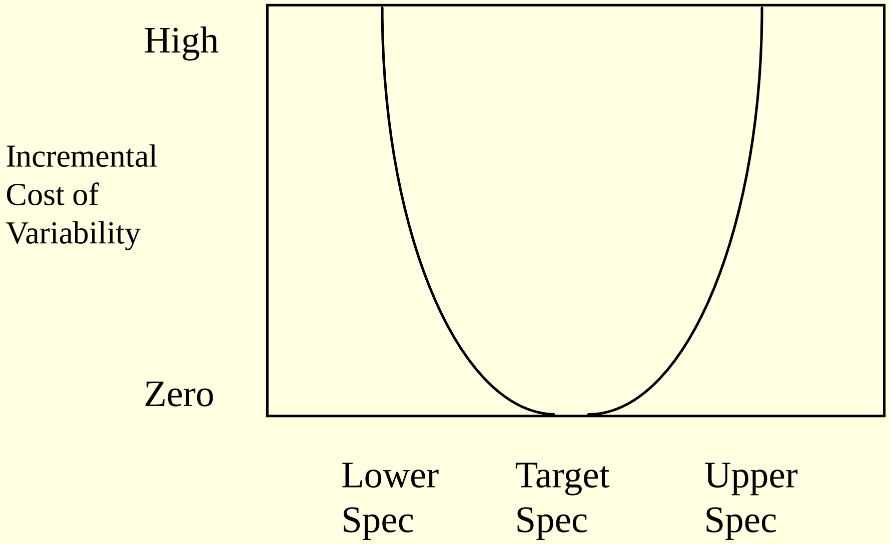
Example: A molding process that always leaves "burrs" or flaws on a molded item.

Taguchi's View of Variation

Traditional view is that quality within the LS and US is good and that the cost of quality outside this range is constant, where Taguchi views costs as increasing as variability increases, so seek to achieve zero defects and that will truly minimize quality costs.



Traditional View



Taguchi's View

Process Capability

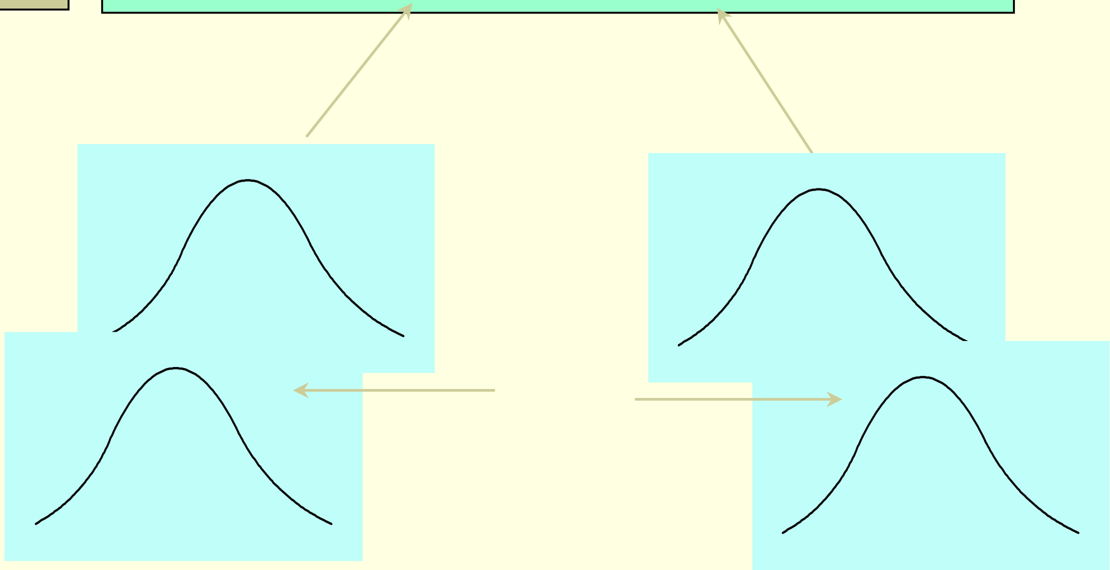
- Process limits
- Specification limits
- How do the limits relate to one another?

Process Capability Index, C_{pk}

Capability Index shows how well parts being produced fit into design limit specifications.

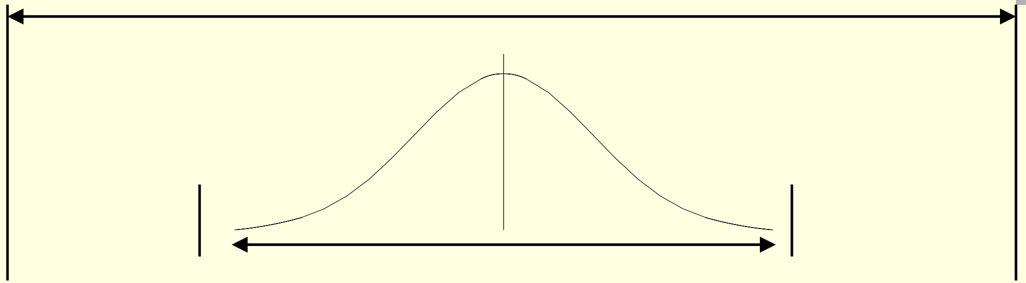
$$C_{pk} = \min\left(\frac{\overline{X} - LTL}{3\sigma} \text{ or } \frac{UTL - \overline{X}}{3\sigma} \right)$$

As a production process produces items small shifts in equipment or systems can cause differences in production performance from differing samples.



Shifts in Process Mean

Process Capability – A Standard Measure of How Good a Process Is.



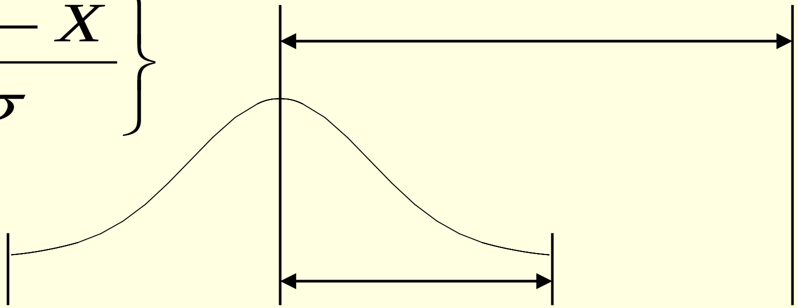
A simple ratio:

$$\frac{\text{Specification Width}}{\text{Actual “Process Width”}}$$

Generally, the bigger the better.

Process Capability

$$C_{pk} = Min\left\{ \frac{\bar{X} - LTL}{3\sigma} ; \frac{UTL - \bar{X}}{3\sigma} \right\}$$



- This is a “one-sided” Capability Index
- Concentration on the side which is closest to the specification - closest to being “bad”

Coffee Bean Bag Example

- Ron Valdez Café roasts and nationally distributes 16 ounce bags of coffee beans. The Omaha World Herald has just published an article that claims that the bags frequently have less than 15 ounces of coffee beans in out 16 ounce bags.
- Let's assume that the government says that we must be within ± 5 percent of the weight advertised on the bag.
- Upper Tolerance Limit = 16 + .05(16) = 16.8 ounces
- Lower Tolerance Limit = 16 – .05(16) = 15.2 ounces
- We go out and buy 1,000 bags of coffee beans and find that they weight an average of 16.103 ounces with a standard deviation of .347 ounces.

Cereal Box Process Capability

- Specification or Tolerance Limit
  - Upper Spec = 16.8 oz
  - Lower Spec = 15.2 oz
- Observed Weight
  - Mean = 16.103 oz
  - Std Dev = .347 oz

$$C_{pk} = Min\left\{\frac{\overline{X} - LTL}{3\sigma};\frac{UTL - \overline{X}}{3\sigma}\right\}$$

$$C_{pk} = Min\left\{\frac{16.103 - 15.2}{3(.347)};\frac{16.8 - 16.103}{3(.347)}\right\}$$

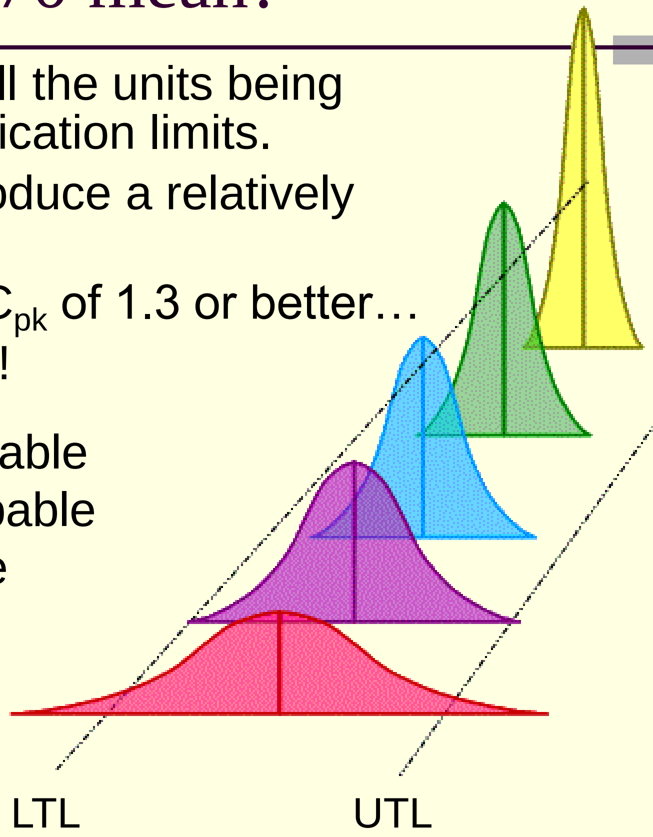
$$C_{pk} = Min\{.867; .670\}$$

$$C_{pk} = .670$$

What does a C_{pk} of .670 mean?

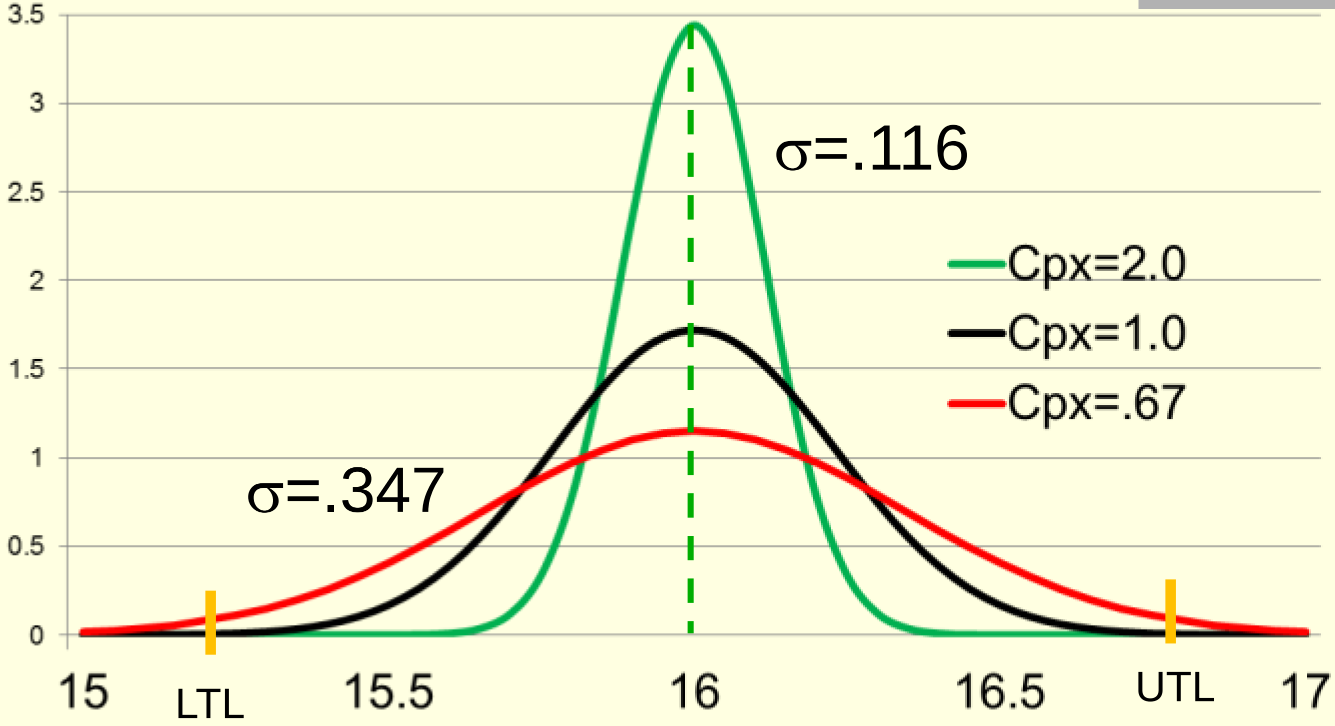
- An index that shows how well the units being produced fit within the specification limits.
- This is a process that will produce a relatively high number of defects.
- Many companies look for a C_{pk} of 1.3 or better… 6-Sigma company wants 2.0!

Cpk < 1 The process is not capable
Cpk = 1 The process is just capable
Cpk > 1 The process is capable



C_{pk}	σ	DPMO
2.00	6.0	3.4
1.67	5.0	233
1.33	4.0	6,210
1.00	3.0	66,800
0.67	2.0	308,540

Cpx comparison



Types of Statistical Sampling

- **Attribute (Go or no-go information)**
  - **Defectives refers to the acceptability of product across a range of characteristics.**
  - **Defects refers to the number of defects per unit which may be higher than the number of defectives.**
  - ***p*-chart application**

Types of Statistical Sampling

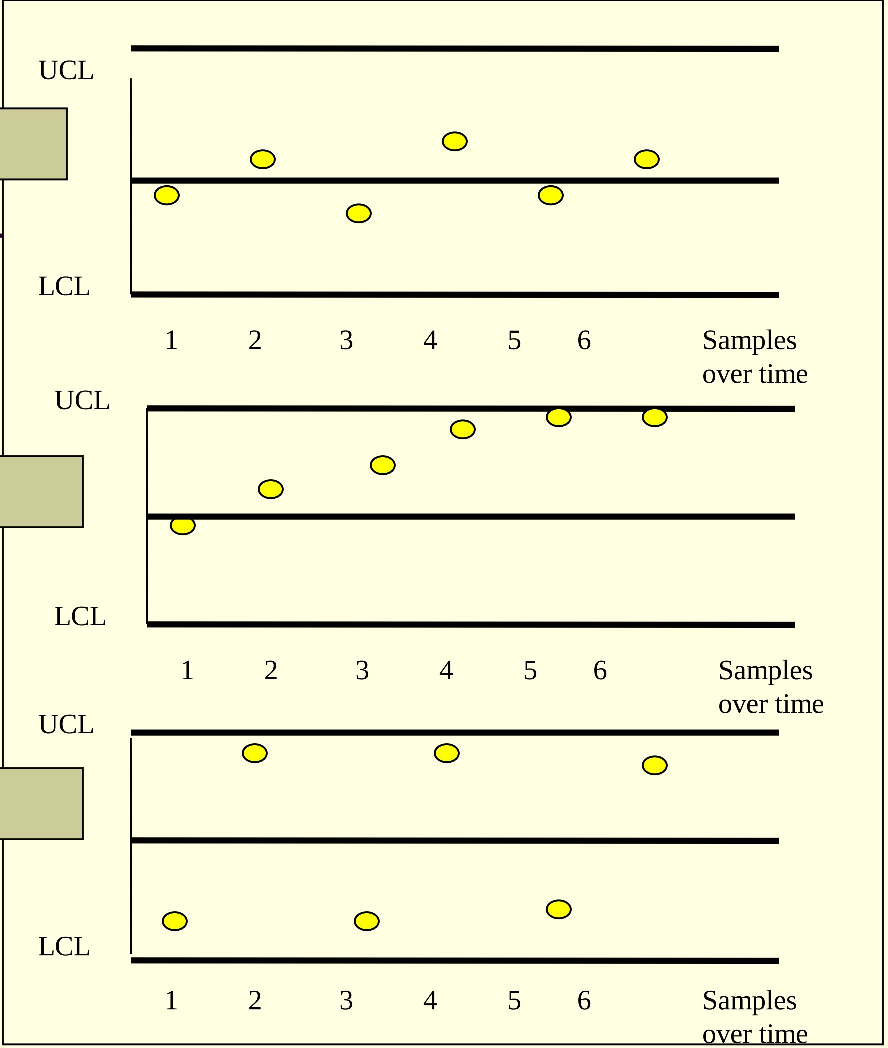
- **Variable (Continuous)**
  - **Usually measured by the mean and the standard deviation.**
  - **X-bar and R chart applications**

Statistical Process Control (SPC) Charts

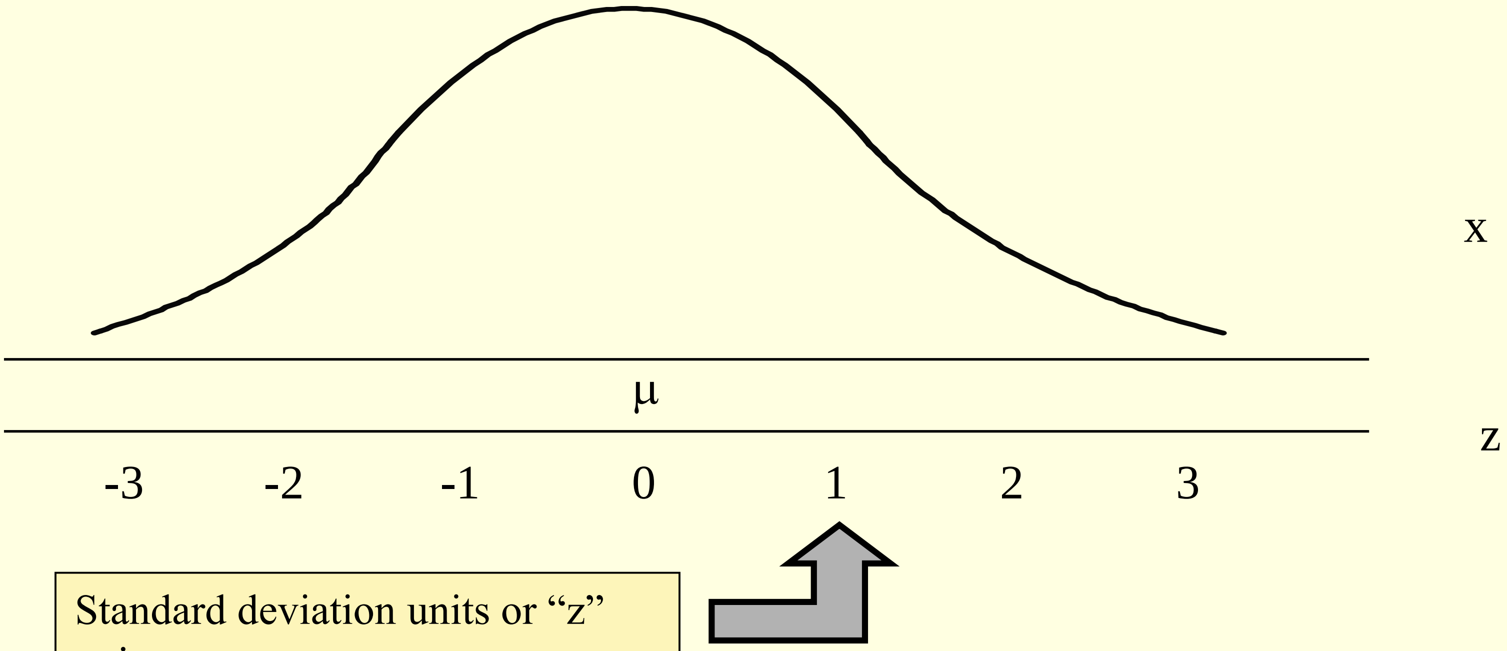
Normal Behavior

Possible problem, investigate

Possible problem, investigate



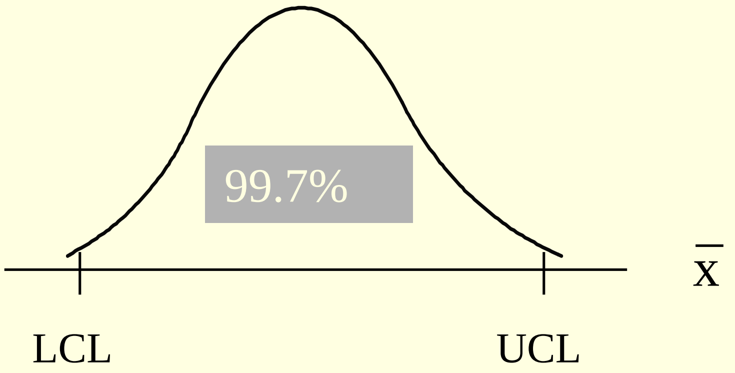
Control Limits are based on the Normal Curve



Standard deviation units or “z” units.

Control Limits

We establish the Upper Control Limits (UCL) and the Lower Control Limits (LCL) with plus or minus 3 standard deviations from some x-bar or mean value. Based on this we can expect 99.7% of our sample observations to fall within these limits.



Define the problem

CTQ – Critical to Quality

The number of chocolate chips in each cookie.

Define a consistent way to count the chocolate chip.

Sample Information

Cookie

Sample	1	2	3	4	5	$\bar{x}$	Range	Defective
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								

n=

$\sum \bar{x} =$

$\sum R =$

$\bar{\bar{x}} =$

$\bar{R}=$

Statistical Process Control Formulas: Attribute Measurements (*p*-Chart) – binomial based

Sample size: n

Proportion defective $$\bar{p} = \frac{\text{Total Number of defective units from all samples}}{\text{Number of samples} \times \text{Sample size(n)}}$$

Standard error $$s_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Compute control limits:

$$UCL = \bar{p} + z\, s_p$$

$$LCL = \bar{p} - z\, s_p$$

Example of Constructing a *p*-chart:

Calculate the average of the sample proportions

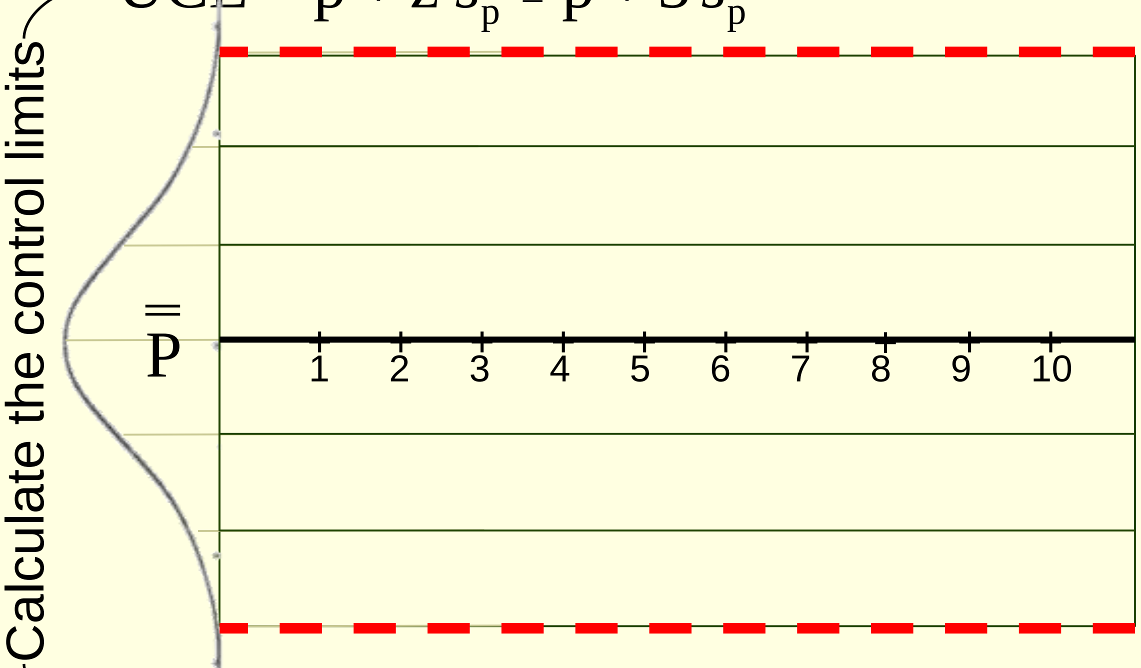
$$\bar{p} = \frac{\quad}{\quad} =$$

Calculate the standard deviation of the sample proportion

$$s_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{\quad (1- \quad)}{\quad}} =$$

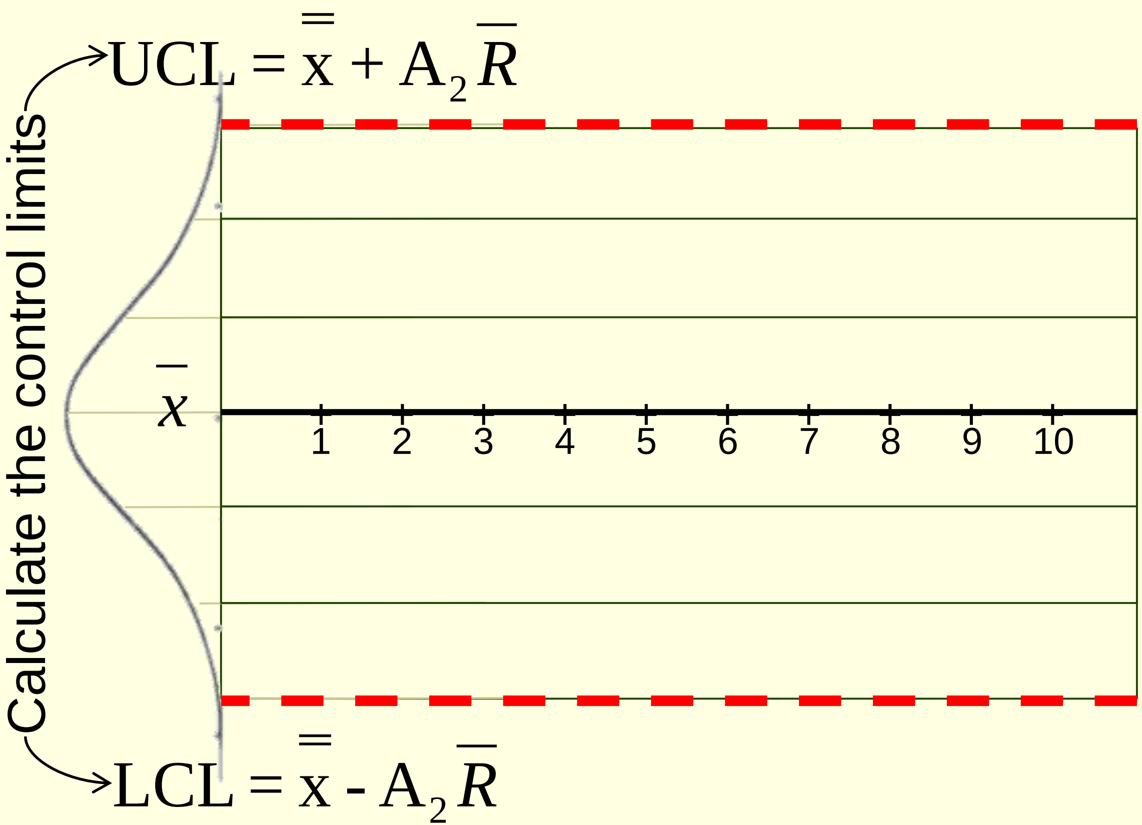
Constructing a *p*-chart

$$UCL = \bar{p} + z\, s_p = \bar{p} + 3\, s_p$$



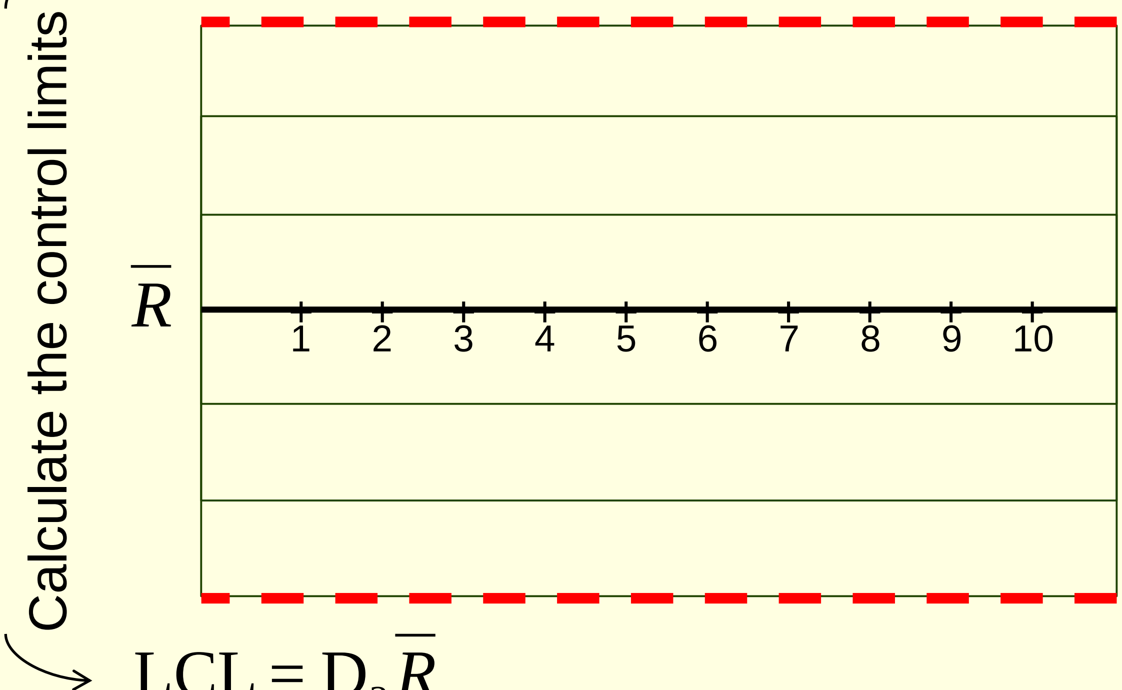
$$LCL = \bar{p} - z\, s_p = \bar{p} - 3\, s_p$$

Constructing a Xbar chart



Constructing an R chart

$$UCL = D_4\bar{R}$$



$$LCL = D_3\bar{R}$$

Basic Forms of Statistical Sampling for Quality Control

- **Acceptance Sampling is sampling to accept or reject the immediate lot of *product* at hand**
- **Statistical Process Control is sampling to determine if the process is within acceptable limits**

Acceptance Sampling

- Purposes
  - Determine quality level
  - Ensure quality is within predetermined level
- Advantages
  - Economy
  - Less handling damage
  - Fewer inspectors
  - Upgrading of the inspection job
  - Applicability to destructive testing
  - Entire lot rejection (motivation for improvement)

Acceptance Sampling (Continued)

- Disadvantages
  - Risks of accepting “bad” lots and rejecting “good” lots
  - Added planning and documentation
  - Sample provides less information than 100-percent inspection

Acceptance Sampling:
Single Sampling Plan

A simple goal

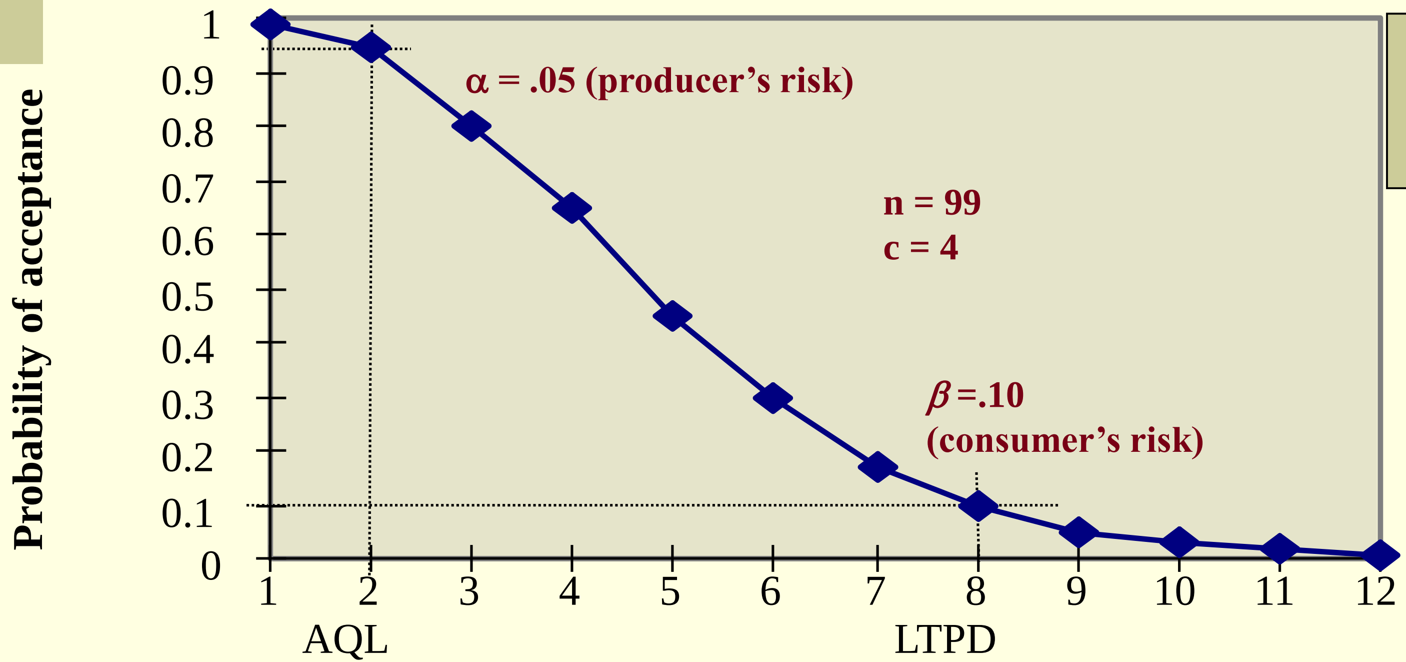
Determine (1) how many units, *n*, to sample from a lot, and (2) the maximum number of defective items, *c*, that can be found in the sample before the lot is rejected

Risk

- Acceptable Quality Level (AQL)
  - Maximum acceptable percentage of defectives defined by producer.
- The α (Producer's risk) is the probability of rejecting a good lot
- Lot Tolerance Percent Defective (LTPD)
  - Percentage of defectives that defines consumer's rejection point.
- The β (Consumer's risk) is the probability of accepting a bad lot

Operating Characteristic Curve

The OCC brings the concepts of producer's risk, consumer's risk, sample size, and maximum defects allowed together



Acceptable Quality Level

Lot Tolerance Percent defective

The shape or slope of the curve is dependent on a particular combination of the four parameters

Well Equipped Traveler

Acceptance Sampling Problem

Well Equipped Traveler, a manufacturer of designer travel accessories, provides Passport/ID pouches to Saks Inc. and its subsidiaries: Saks Fifth Avenue, Herberger's, Younkers, Parisian, Carson Pirie Scott, Boston Store, Bergner's, Loveman's, Parks-Belk, and Brody's. Well Equipped Traveler has set an Acceptable Quality Level of 1% and accepts a 5% risk of lots being rejecting at or below this level. Saks Inc. considers lots with 3% defectives to be unacceptable and will assume a 10% risk of accepting a defective lot.

Develop a sampling plan for Saks Inc. and determine a rule for the receiving inspection personnel to follow.

Example: Step 1. What is given and what is not?

In this problem,

Acceptable Quality Level, **AQL = 0.01**

Tolerance Percent Defective, **LTPD = 0.03**

Producer's risk, **α = 0.05**

Consumer's risk, **β = 0.10**

To determine your sampling plan you need:

c *(the critical number of defects)* and,

n *(the sample size).*

Example: Step 2. Determine "c"

First divide LTPD by AQL.

$$\frac{LTPD}{AQL} = \frac{.03}{.01} = 3$$

Then find the value for "c" by selecting the value in the appropriate table "n AQL" column that is equal to or just greater than the ratio above.

$$\text{Sample Plan table for } \alpha = 0.05, \beta = 0.10$$

c	*LTPD/AQL*	*n AQL*	*c*	*LTPD/AQL*	*n AQL*
0	44.890	0.052	5	3.549	2.613
1	10.946	0.355	**6**	3.206	3.286
2	6.509	0.818	7	2.957	3.981
3	4.890	1.366	8	2.768	4.695
4	4.057	1.970	9	2.618	5.426

Example: Step 3. Determine Sample Size

Now given the information below, compute the sample size in units
to generate your sampling plan

$$c = 6, \text{ from Table}$$
$$n \cdot AQL = 3.286, \text{ from Table}$$
$$AQL = .01, \text{ given in problem}$$

$$n \cdot AQL / AQL = 3.286 / .01 = 328.6$$
$$\Rightarrow 329 \text{ (always round up)}$$

Sampling Plan:

Take a random sample of 329 units from a lot.

Reject the lot if *more than* 6 units are defective.