Chapter 8
Interval Estimation

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t Distribution

- ✓ The *t distribution* is a family of similar probability distributions.
- ✓ A specific *t* distribution depends on a parameter known as the _____.
- ✓ As the number of degrees of freedom increases, the difference between the *t* distribution and the standard normal probability distribution becomes smaller and smaller.
- ✓ A *t* distribution with more degrees of freedom has less dispersion.
- ✓ The mean of the *t* distribution is zero.

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Interval Estimation of a Population Mean: Small-Sample Case ($n < 30$) with σ Unknown

✓ The interval estimate is given by:

$$\bar{x} \pm t \cdot s_{\bar{x}} \qquad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Where   1-α   is the confidence coefficient

α/2   is the *t* value providing an area of
α/2 in the upper tail of a *t* distribution
with *n* - 1 degrees of freedom

s   is the sample standard deviation

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Example: Apartment Rents

A reporter for a student newspaper is writing an article on the cost of off-campus housing. A sample of 10 one-bedroom units within a half-mile of campus resulted in a sample mean of $350 per month and a sample standard deviation of $30.

Let us provide a 95% confidence interval estimate of the mean rent per month for the population of one-bedroom units within a half-mile of campus. We'll assume this population is normally distributed.

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Example: Apartment Rents

✓ t Value

At 95% confidence, $1 - \alpha = .95$, $\alpha = .05$, and $\alpha/2 = .025$.

$t_{.025}$ is based on $n - 1 = 10 - 1 = 9$ degrees of freedom. In the t distribution table we see that $t_{.025} = 2.262$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	0.025	.01	.005
.	.	.	.	.	.
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
.	.	.	.	.	.

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Example: Apartment Rents

✓ Interval Estimation of a Population Mean:
Small-Sample Case ($n < 30$) with σ Unknown

$$\bar{x} \pm t_{.025} \frac{s}{\sqrt{n}}$$

We are 95% confident that the average rent per month for the population of one-bedroom units within a half-mile of campus is between $328.54 and $371.46

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Example: Apartment Rents

✓ Interval Estimation of a Population Mean:
  Small-Sample Case ($n < 30$) with σ Unknown

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \qquad\qquad \bar{x} \pm t \cdot s_{\bar{x}}$$

$$\boxed{P(\$\quad \leq \mu \leq \$\quad) = .95}$$

We are 95% confident that the average rent per month for the population of one-bedroom units within a half-mile of campus is between $328.54 and $371.46

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Sample Size for an Interval Estimate of a Population Mean

✓ Let E = the maximum sampling error mentioned in the precision statement.

✓ E is the amount added to and subtracted from the point estimate to obtain an interval estimate.

✓ E is often referred to as the <u>margin of error</u>.

✓ We have $\qquad\qquad E = z\frac{\sigma}{\sqrt{n}}$

✓ Solving for n we have $\qquad \boxed{n = \frac{z^2\sigma^2}{E^2}}$

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Using the **t** table to find **z** values

Degrees of Freedom	Area in Upper Tail				
	.10	.05	0.025	.01	.005
.	.	.	.	.	.
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
infinity	1.282	1.645	1.960	2.326	2.576

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Example: The Buckle, Inc.

Suppose that Buckle's management team wants an estimate of the population mean income such that there is a .95 probability that the sampling error is $500 or less.

How large a sample size is needed to meet the required precision?

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Example: The Buckle, Inc.

✓ Sample Size for Interval Estimate of a Population Mean

$$n = \frac{z^2\sigma^2}{E^2}$$

At 95% confidence,
$z_{.025} = 1.96$, E=$500,
$\sigma = 5000$ (recall from past example)

$$n = \frac{(1.96)^2(5,000)^2}{(500)^2} = 384.16 \approx 385$$

We need to sample 385 to reach a desired precision of $\pm$ $500 at 95% confidence.

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Example: Political Science, Inc.

✓ Three rules for estimating a Population Standard Deviation
1. Use σ from another similar study.
2. Use s from a pilot study, let $s \cong \sigma$

3. Let $\sigma \cong \dfrac{range}{6}$

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Interval Estimation
of a Population Proportion

✓ The interval estimate is given by:

$$\bar{p} \pm z \cdot s_{\bar{p}} \qquad s_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Where

$1 - \alpha$ is the confidence coefficient

z is the z value providing an area of $\alpha/2$ in the upper tail of the
standard normal probability distribution is the sample proportion $\bar{p}$

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Example: Political Science, Inc.

Political Science, Inc. (PSI) specializes in voter polls and surveys designed to keep political office seekers informed of their position in a race.   Using telephone surveys, interviewers ask registered voters who they would vote for if the election were held that day.

In a recent election campaign, PSI found that 220 registered voters, out of 500 contacted, favored a particular candidate.   PSI wants to develop a 95% confidence interval estimate for the proportion of the population of registered voters that favors the candidate.

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Example: Political Science, Inc.

✓ Interval Estimate of a Population Proportion

$$s_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad \text{where} \quad n = 500$$
$$z = \qquad \bar{p} =$$

$$\bar{p} \pm z \cdot s_{\bar{p}}$$

PSI is 95% confident that between 39.65% and 48.35% of all voters that favors the candidate.

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Sample Size for an Interval Estimate of a Population Proportion

✓ Let E = the maximum sampling error mentioned in the precision statement.

✓ We have
$$E = z \cdot \sqrt{\frac{p(1-p)}{n}}$$

Solving for n we have
$$n = \frac{z^2 \cdot p(1-p)}{E^2}$$

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Example: Political Science, Inc.

Suppose that PSI would like a .99 probability that the sample proportion is within + .03 of the population proportion. A similar study performed on the same candidate in the same region resulted in a sample proportion of 44%.

How large a sample size is needed to meet the required precision?

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Example: Political Science, Inc.

✓ Three rules for estimating a Population Proportion
  1. Use p from another similar study.
  2. If no estimate of p is available, set p to .50
  3. Use $\bar{p}$ from a pilot study, let $\bar{p} \cong p$

✓ Sample Size for Interval Estimate of a Population Proportion
  At 99% confidence, $z_{.005} = 2.576$. Note, .44 is the best estimate of p in the above expression.
✓ If no information is available about p, what would you recommended as the size of $n?$

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In class exercise – Kearney 6th Graders

Each member of a random sample of 25 sixth-graders in Kearney keeps a record for one week of the amount of time spent watching television. The sample mean and sample standard deviation computed from the results are 15 hours and 9 hours respectively. Construct a 95% confidence interval for the population mean.

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