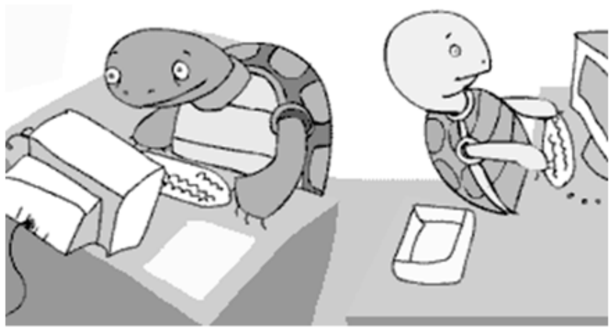
Chapter 10



Pssst! Coffee helps!

Sample Size for an Interval Estimate of a Population Proportion

Let E = the maximum sampling error mentioned in the precision statement.

We have $E = z_{\alpha/2}\sqrt{\dfrac{p(1-p)}{n}}$

Solving for n we have $$n = \frac{(z_{\alpha/2})^2\, p(1-p)}{E^2}$$

Example: Political Science, Inc.

Suppose that PSI would like a .99 probability that the sample proportion is within + .03 of the population proportion. A similar study performed on the same candidate in the same region resulted in a sample proportion of 44%.

How large a sample size is needed to meet the required precision?

Example: Political Science, Inc.

- ✓ Three rules for estimating a Population Proportion
  1. Use ***p*** from another similar study.
  2. If no estimate of ***p*** is available, set ***p*** to .50
  3. Use $\overline{p}$ from a pilot study, let $\overline{p} \cong p$

$$n = \frac{(z_{\alpha/2})^2 p(1-p)}{E^2} =$$

- ✓ Sample Size for Interval Estimate of a Population Proportion
  At 99% confidence, $z_{.005} = 2.576$. Note, .44 is the best estimate of ***p*** in the above expression.
- ✓ If no information is available about ***p***, what would you recommended as the size of ***n?***

In class exercise – Kearney 6th Graders

- ✓ Each member of a random sample of 25 sixth-graders in Kearney keeps a record for one week of the amount of time spent watching television. The sample mean and sample standard deviation computed from the results are 15 hours and 6 hours respectively. Construct a 95% confidence interval estimate for the population mean. Assume that the the time students spend watching television is normally distributed.

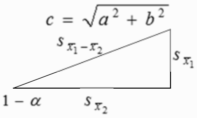
Chapter 10
Statistical Inference - Two Populations

- ✓ Estimation of the Difference between the Means of Two Populations, Large Sample
  – Independent Samples
  – Population variances are unknown and unequal

Interval Estimate for the difference between two population means – large sample, n>30

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \cdot s_{\bar{x}_1 - \bar{x}_2}$$

$c = \sqrt{a^2 + b^2}$



Where $s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Interval Estimate $P(\ldots \le \mu_1 - \mu_2 \le \ldots) = 1 - \alpha$

For example $P(123 \le \mu_1 - \mu_2 \le 456) = .99$

Example – Difference of Two Means

At Carter Ironworks, two machines are used to produce metal rods. A random sample of 49 rods from Machine 2 and a random sample of 36 rods from Machine 1 give these results with respect to the lengths of metal rods produced.

$\bar{x}_2 = 5.91 \quad s_2^2 = 0.018 \quad \bar{x}_1 = 6.01 \quad s_1^2 = 0.020$

Construct the 95% confidence interval estimate for $\mu_1 - \mu_2$

Carter Ironworks is 95% confident that the average length of rods from machine #1 is 0.04' to 0.16 feet longer that rods from machine #2.

Two Population Proportions

- ✓ Estimation of the Difference between the Means of Two Proportions, Large Sample
  - – Independent Sample

Interval Estimate for the difference between two population proportions – large sample

$$(\bar{\rho}_1 - \bar{\rho}_2) \pm z_{\alpha/2} \cdot s_{\bar{p}_1 - \bar{p}_2}$$

Where $s_{\bar{p}_1 - \bar{p}_2} = \sqrt{s^2_{\bar{p}_1} + s^2_{\bar{p}_2}} = \sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{n_1} + \frac{\bar{p}_2(1 - \bar{p}_2)}{n_2}}$

Interval Estimate $P(\ldots \le \rho_1 - \rho_2 \le \ldots) = 1 - \alpha$

For example $P(.04 \le \rho_1 - \rho_2 \le .05) = .99$

Example – Difference of Two Proportions

Political Science, Inc. (PSI) performed a telephone survey asking registered voters who they would vote for if the election were held that day.

PSI found that 384 registered voters, out of 800 contacted, favored a particular candidate. The previous week showed 220 out of 500 voters favored the candidate. PSI wants to develop a 80% confidence interval estimate for the change in proportion of all registered voters that favors the candidate.`

Current	**Previous Week**
$\bar{p}_1 = \frac{384}{800} = .48$	$\bar{p}_2 = \frac{220}{500} = .44$
$n_1 = 800$	$n_2 = 500$

$$(\bar{\rho}_1 - \bar{\rho}_2) \pm z_{\alpha/2} \cdot s_{\bar{p}_1 - \bar{p}_2}$$

$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{n_1} + \frac{\bar{p}_2(1 - \bar{p}_2)}{n_2}} = \sqrt{\frac{.48 \times .52}{800} + \frac{.44 \times .56}{500}}$$

We are 80% confident that the candidate's approval increased between .4% and 7.6%.

In class exercise

Each member of a random sample of 50 sixth-graders in Kearney kept a record for one week of the amount of time spent watching television. The sample mean and sample variance are 15 hours and 10 hours^2.

A second random sample of 40 second-graders in the Greenhill school district also kept records for one week of the amount of time spent watching television. The sample mean and sample variance are 10 hours and 5 hours^2.

Construct a 95% confidence interval for the mean difference between Kearney 6th graders and Greenhill 2nd graders.

Kearney & Greenhill

Kearney 6th graders	Greenhill 2nd graders
$n_1 = 50$, $\bar{x}_1 = 15$ *hours*	$n_2 = 40$, $\bar{x}_2 = 10$ *hours*
$s_1^2 = 10$ *hours*^{2}	$s_2^2 = 5$ *hours*^{2}

In class exercise

A Gallup poll found that 16% of 505 men and 25% of 496 women surveyed favored a law forbidding the sale of all beer, wine, and liquor throughout the nation. Develop a 95% confidence interval for the difference between the proportion of women who favor such a ban and the proportion of men who favor such a ban.