Chapter 9
Hypothesis Testing

"Its not my fault when you consider that my three husbands have had twenty wives."
– Ava Gardner, on being asked about her failed marriages to Mickey Rooney, Artie Shaw, and Frank Sinatra

Men are generally more careful of the breed of their horses and dogs than of their children
- William Penn

Example: NHL

The NHL currently has a rigorous (or so they claim) drug testing policy for all players. The NHL follows the testing "innocent until proven guilty approach"

H_o: The player does not take steroids
H_a: The player does take steroids

Type I and Type II Errors

- ✓ Since hypothesis tests are based on sample data, we must allow for the possibility of errors.
- ✓ A Type I error is rejecting H_0 when it is true.
  - α (alpha) is the probability of rejecting the truth
  - α is called the level of significance. The person conducting the hypothesis test specifies the maximum allowable probability of making a Type I error
- ✓ A Type II error is failing to reject H_0 when it is false.
  - β (beta) is the probability of failing to reject what is false

Example: NHL Drug Testing

H_o: The player does not take steroids
H_a: The player does take steroids

	Population Condition	
Conclusion	H_o True	H_a True
Fail to Reject H_0		
Reject H_0		

NHL Example

Given: a certain player has recently taken steroids specifically for muscular enhancement.

What type of error does this player hope will be made when he is tested for steroids?

What type of error does the NHL what to avoid?

Example: Omaha EMS

Omaha provides one of the most comprehensive emergency medical services in the world. Operating in a multiple hospital system with approximately 10 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 6 minutes or less.

The director of medical services wants to formulate a hypothesis test (95% confidence) that could use a sample of emergency response times to determine whether or not the service goal of 6 minutes or less is being achieved.

- ✓ One-Tailed Test about a Population Mean

Let $n = 40$, $\bar{x} = 7.25$ minutes, $s = 3.2$ minutes

Example: Omaha EMS

Hypotheses	Conclusion and Action
H_0: $\mu \leq$ ____	The emergency service is meeting the response goal; no follow-up action is necessary.
H_a: $\mu >$ ____	The emergency service is not meeting the response goal; appropriate follow-up action is necessary.
Where	μ = mean response time for the population of medical emergency requests.

Example: Omaha EMS

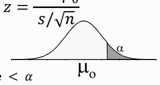
	Population Condition	
Conclusion	H_o True ($\mu \leq 6$)	H_a True ($\mu > 6$)
Fail to Reject H_0 (Conclude $\mu \leq 6$)	Correct Conclusion	Type II Error
Reject H_0 (Conclude $\mu > 6$)	Type I Error	Correct Conclusion

The 7 Steps of Hypothesis Testing

1. Determine the appropriate hypotheses.
2. Select the test statistic and its distribution
3. Specify the level of significance α for the test.
4. Develop the decision rule for rejecting H_0.
5. Collect the sample data and compute the value of the test statistic.
6. Statistical Decision
   - Compute the p -value based on the test statistic and compare it to α, to determine whether or not to reject H_0.
7. Interpretation

One-Tailed Tests About a Population Mean: Large-Sample Case ($n \geq 30$)

1. Hypotheses H_0: $\mu \leq 6$
   H_a: $\mu > 6$
2. Test Statistic:
   If σ Known, $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ if σ Unknown $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
3. Specify α
   α = .05
4. Rejection Rule:



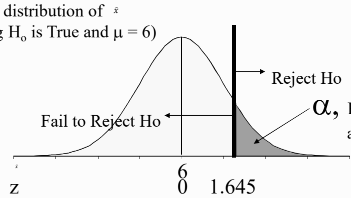
Reject Ho if P−value $< \alpha$
$\bar{x}$ is greater than μ_o

Example: Omaha EMS

✓ One-Tailed Test about a Population Mean

Let α = P (Type I Error) = .05

Sampling distribution of $\bar{x}$
(assuming H_o is True and μ = 6)

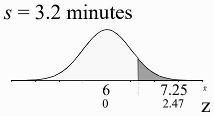


Example: Omaha EMS

✓ One-Tailed Test about a Population Mean

Let n = 40, $\bar{x}$ = 7.25 minutes, s = 3.2 minutes

5. $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.25 - 6}{3.2/\sqrt{40}} =$



The Use of p -Values

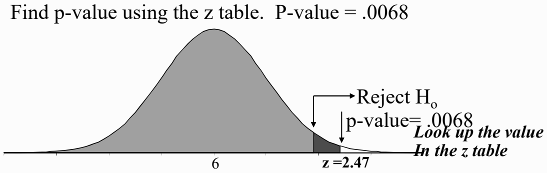
- ✓ The p -value is the probability of obtaining a sample result that is at least as unlikely as what is observed.
- ✓ The p -value can be used to make the decision in a hypothesis test by noting that:
  - if the p -value is less than the level of significance α, the value of the test statistic is in the rejection region.
  - if the p -value is greater than or equal to α , the value of the test statistic is not in the rejection region.
- ✓ **Reject H_0 if the p-value < α.**

Example: Omaha EMS

✓ Using the p-value to Test the Hypothesis

Recall that z =2.47 for $\bar{x}$ =7.25. α = .05

Find p-value using the z table. P-value = .0068



Since p-value < α, that is .0068<.05, we reject H_o $and\ \bar{x}\ is\ greater\ than\ 6\ (\mu_0)$

Example: Omaha EMS

7. <u>Conclusion</u>**:** We are 95% confident that Omaha EMS is not meeting the response goal of 6 minutes; appropriate action should be taken to improve service.

Tests About a Population Mean: Small-Sample Case ($n < 30$)

- ✓ Test Statistic where σ is unknown

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

This test statistic has a *t* distribution with *n* - 1 degrees of freedom.

H_0: $\mu \le \mu_0$ One Tailed

H_0: $\mu \ge \mu_0$ One Tailed

H_0: $\mu = \mu_0$ Two Tailed

p -Values and the *t* Distribution

- ✓ The format of the *t* distribution table provided in most statistics textbooks does not have sufficient detail to determine the exact *p* -value for a hypothesis test.
- ✓ However, we can still use the *t* distribution table to identify a range for the *p* -value.
- ✓ An advantage of computer software packages is that the computer output will provide the *p* -value for the *t-d*istribution.

NVIDIA Example

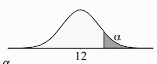
NVIDIA has announced the latest turning architecture processor that computes 16 trillion floating-point operations per second, 500 trillion tensor operations per second, 10 GigaRays per second. Internally to the company, the design specification required that the population average speed for all processors be greater than 12 GigaRays. From the Quality Control department perspective, is the design specification met with 99% confidence? (the population is normally distributed)

A random sample of 21 chips gives a sample average of 12.27 GRays with a sample standard deviation of 0.80 GRays.

Research hypothesis

Using the 7 Steps of Hypothesis Testing

1. $H_o: \mu \leq$ ____ $\quad H_A: \mu >$ ____
2. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
3. $\alpha = .01$
4. Rejection Rule:



Reject Ho if $P\text{-value} < \alpha$
and $\bar{x} > 12$

Using the p-value approach

5. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} =$

6. Statistical Decision

____ $< 1.55 <$ ____
____ $< p - value <$ ____

p-value = .06882 (Using Excel)

Reject H_0 if p-value < α,
Since $.06882 \not< .05$, fail to reject H_o

Conclusion

7. At 99% confidence, the data is not sufficient to prove that the average speed for all processors is greater than 12 GigaRays before failure.

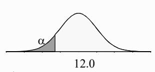
NVIDIA Example

- ✓ Loper Technologies (LT), as a major client, also receives exactly the same sample information from Intel. From LT's perspective, can the design specification be refuted with 99% confidence? (use at least an average speed of 12 GigaRays)
- ✓ A random sample of 21 chips gives a sample average of 12.27 GigaRays with a sample standard deviation of 0.03 GigaRays.

Testing a claim

Using the 7 Steps of Hypothesis Testing

1. $H_o: \mu \geq 12.00$ $H_A: \mu < 12.00$
2. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
3. $\alpha = .01$
4. Rejection Rule:

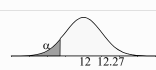


Reject Ho if P−value $< \alpha$
and $\bar{x} < 12.00$

Using the 7 Steps of Hypothesis Testing

6. Statistical Decision

Fail to reject Ho
$\bar{x}$ is not less than 12.00



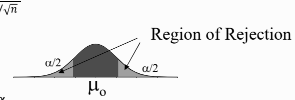
7. At 99% confidence, the data is not sufficient to refute that the average speed for all processors is at least 12 GigaRays before failure.

-OR-

At 99% confidence, the data is not sufficient to prove that the average speed for all processors is less than 12 GigaRays before failure.

Two-Tailed Tests About a Population Mean:
Large-Sample Case ($n \geq 30$)

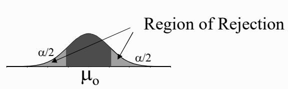
- ✓ Hypotheses: $H_o: \mu = \mu_o$ $H_a: \mu \neq \mu_o$
- ✓ Test Statistic:
  - σ Known $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
  - σ Unknown $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- ✓ Rejection Rule:



Reject Ho if P−value $< \alpha$
$\bar{x}$ may fall on the either side of μ_o

Two-Tailed Tests About a Population Mean:
Small-Sample Case ($n < 30$)

- ✓ Hypotheses: $H_o: \mu = \mu_o$ $H_a: \mu \neq \mu_o$
- ✓ Test Statistic:
  - σ Unknown $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- ✓ Rejection Rule:



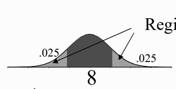
Reject Ho if P−value $< \alpha$
$\bar{x}$ may fall on the either side of μ_o

Loper Electronics

- ✓ Loper Openair U-acoustic Devices (LOUD) manufactures flat panel speakers. The average resistance per speaker must be exactly 8 ohms. The distributor of LOUD speakers will reject a shipment of speakers if the average resistance is not exactly 8 ohms. A sample of 25 speakers yields an average resistance of 8.15 ohms with a standard deviation of .5 ohms.
- ✓ Should the distributor keep or return the shipment? Let $\alpha = .05$.

Using the 7 Steps of Hypothesis Testing

1. $H_o: \mu = 8 \qquad H_A: \mu \neq 8$
2. $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
3. $\alpha = .05$
4. Rejection Rule:



Reject Ho if P−value $< \alpha$
$\bar{x}$ may fall on the either side of μ_o

Using the 7 Steps of Hypothesis Testing

5. $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{8.15 - 8.00}{.5/\sqrt{25}} = 1.5$
6. Statistical Decision
   1.318<1.5<1.711
   .10 < P-value < .20

Computation of p-value for a two-tailed test Step 3. See page 353(10th), page 364 (11th), 398 (12th), 401 (13th)
- Double the tail area, or probability to obtain the p-value

Use the Excel function T.DIST.2T to obtain the true value.
P-value = .146656

Fail to reject Ho

Using the 7 Steps of Hypothesis Testing

7. The data is not sufficient to prove that the population average resistance of all the speakers is not 8 ohms.
   - Keep the shipment

A Summary of Forms for Null and Alternative Hypotheses about a Population Proportion

- ✓ The equality part of the hypotheses always appears in the null hypothesis.
- ✓ In general, a hypothesis test about the value of a population proportion p must take one of the following three forms (where p_0 is the hypothesized value of the population proportion).

$$Ho: \rho \geq \rho_o \qquad Ho: \rho \leq \rho_o \qquad Ho: \rho = \rho_o$$
$$Ha: \rho < \rho_o \qquad Ha: \rho > \rho_o \qquad Ha: \rho \neq \rho_o$$

Tests About a Population Proportion: Large-Sample Case ($np \geq 5$ and $n\,(1 - p) \geq 5$)

✓ Test Statistic: $z = \frac{\bar{p} - p_0}{\sigma_p}$ where $\sigma_p = \sqrt{\frac{p_0(1-p_0)}{n}}$

H_0: $p \leq p_0$ One-Tailed

H_0: $p \geq p_0$ One-Tailed

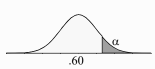
H_0: $p = p_0$ Two-Tailed

Example : UNK Parking

- ✓ UNK is considering building a parking garage for students. The administration believes that more than 60% of the students drive cars to campus. If in a random sample of 250 students, 165 indicate that they drive a car to school is the administration's position supported? Let $\alpha = 0.05$.

Using the 7 Steps of Hypothesis Testing

1. $H_o: \rho \leq .60$ $H_A: \rho > .60$
2. $z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}$ $\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$
3. $\alpha = .05$
4. Rejection Rule:



Reject Ho if P−value $< \alpha$
$\bar{\rho}$ is greater than .60

Using the 7 Steps of Hypothesis Testing

5. $\bar{\rho} = \frac{165}{250} = .66$ $\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.6 \times .4}{250}} = .03098$

   $z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{.66 - .60}{.03098} = 1.936$

6. Statistical Decision
   P-value = .0262
   .0262 < .05, Reject Ho, Accept Ha
7. The administration is 95% confident that more than 60% of the students drive cars to campus.

Chapter 10

Hypothesis Tests About the Difference
Between the Means of Two Populations: Independent Samples

- ✓ Hypothesis Forms:

  H_0: $\mu_1 - \mu_2 \leq 0$ H_0: $\mu_1 - \mu_2 \geq 0$ H_0: $\mu_1 - \mu_2 = 0$

  H_a: $\mu_1 - \mu_2 > 0$ H_a: $\mu_1 - \mu_2 < 0$ H_a: $\mu_1 - \mu_2 \neq 0$

- ✓ Test Statistic:
  - Large-Sample Case (population variance is unknown)

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

Example: Par, Inc.

Par, Inc. is a manufacturer of golf equipment. Par has developed a new golf ball that has been designed to provide “extra distance.” In a test of driving distance using a mechanical driving device, a sample of Par golf balls was compared with a sample of golf balls made by Rap, Ltd., a competitor. The sample data is below.

	Sample #1 Par, Inc.	Sample #2 Rap, Ltd.
Sample Size	n_1 = 120 balls	n_2 = 80 balls
Mean	$\bar{x}_1$ = 225 yards	$\bar{x}_2$ = 218 yards
Standard Deviation	s_1 = 15 yards	s_2 = 20 yards

✓ Hypothesis Tests About the Difference Between the Means of Two Populations: Large-Sample Case

Can we conclude, using a .01 level of significance, that the mean driving distance of Par, Inc. golf balls is greater than the mean driving distance of Rap, Ltd. golf balls?

μ_1 = mean distance for the population of Par, Inc. golf balls

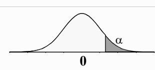
μ_2 = mean distance for the population of Rap, Ltd. golf balls

1. Hypotheses: H_0: $\mu_1 - \mu_2 \leq 0$

   H_a: $\mu_1 - \mu_2 > 0$

Example: Par, Inc.

4. Rejection Rule:



Reject Ho if P−value $< \alpha$

$\bar{x}_1 - \bar{x}_2$ greater than 0.0

5. Calculation

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_o}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(225 - 218) - 0}{\sqrt{\frac{(15)^2}{120} + \frac{(20)^2}{80}}} = \frac{7}{2.62} = 2.67$$

Example: Par, Inc.

6. Statistical Decision

   P-value=.0038

   .0038<.01

   Reject Ho, accept Ha

7. **Conclusion:** We are at least 99% confident that the mean driving distance of Par, Inc. golf balls is greater than the mean driving distance of Rap, Ltd. golf balls.

Hypothesis Testing about p_1 - p_2

- Hypotheses:

$$H_0\!: p_1 - p_2 \leq 0$$
$$H_a\!: p_1 - p_2 > 0$$

- Test statistic:

$$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)_0}{s_{\bar{p}_1 - \bar{p}_2}}$$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} \qquad s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Example: The Israeli – Palestinian Conflict

- Hypothesis Tests about p_1 - p_2

Can we conclude, using a .01 level of significance, that U.S. voters are less resistant to Israeli incursions into Palestinian occupied areas this week than two weeks ago?

p_1 = proportion of the voters resistant the Israeli actions this week.

p_2 = proportion of the voters resistant the Israeli actions two weeks earlier.

$n_1 = 120 \qquad n_2 = 80$

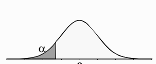
$\bar{p}_1 = .10 \qquad \bar{p}_2 = .22$

Example: The Israeli – Palestinian Conflict

1. Hypotheses: $H_0\!: p_1 - p_2 \geq 0$

   $H_a\!: p_1 - p_2 < 0$

4. Rejection Rule:



Reject Ho if P−value $< \alpha$

$\bar{p}_1 - \bar{p}_2$ is less than 0.0

Example: The Israeli – Palestinian Conflict

5. $\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{120 \times .10 + 80 \times .22}{120 + 80}$ **=.148**

$s_{\bar{p}_1-\bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{.148 \times .852\left(\frac{1}{80}+\frac{1}{120}\right)}$ **=.0512**

$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)_0}{s_{\bar{p}_1-\bar{p}_2}} = \frac{(.10 - .22) - 0}{.0512}$ **= -2.34**

P-Value **= .00964**

Example: The Israeli – Palestinian Conflict

6. Statistical Decision
   Reject H_0 if P-value< α
   .00964 < .01,
   Reject H_o, Accept H_a
7. We are 99% confident that voters are less resistant to the Israeli incursion into occupied Palestinian areas this week than two weeks ago.