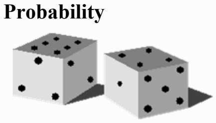
Chapter 4
Introduction to Probability

- **Experiments, the Sample Space, and Counting Rules**
- **Assigning Probabilities to Experimental Outcomes**
- **Events and Their Probabilities**
- **Some Basic Relationships of Probability**
- **Conditional Probability**



An Experiment and Its Sample Space

- ✓ An ______________ is any process that generates well-defined outcomes.
- ✓ The _________________ for an experiment is the **set** of all experimental outcomes.
- ✓ A _________________ is an element of the sample space, any one particular experimental outcome.

Counting Rules

- ✓ Multistep Experiments
- ✓ Factorial, n!
- ✓ Permutation
- ✓ Combination

Counting Rule Multiple-Step Experiments

If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.

Counting Rule - Factorial, n!

$n! = n(n - 1)(n - 2) \ldots (2)(1)$

$0! = 1$

- No replacement
- Use All Observations
- Order Matters

Counting Rule - Factorial, n!

How many orders could 5 students sit in 5 chairs?

5!=5 x 4 x 3 x 2 x 1

No replacement
Use All Observations
Order Matters

Counting Rule - Permutation

$${}_{N}P_{n} = \frac{N!}{(N-n)!}$$

How many orders could 9 students sit in 5 chairs?

$${}_{9}P_{5} = \frac{9!}{(9-5)!} = \frac{9!}{4!}$$

- No replacement
- Use Some Observations
- Order Matters

Counting Rule - Combination

✓ Another useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of *N* objects.

✓ The number of combinations of *N* objects taken *n* at a time is $${}_{N}C_{n} = \frac{N!}{n!(N-n)!}$$

- No replacement
- Use some observations
- Order does not matter

Counting Rule - Combination

$$_NC_n = \frac{N!}{n!(N-n)!}$$

How many combinations could 9 students sit in 5 chairs where order does not matter?

$$_NC_n = \frac{9!}{5!(9-5)!} = \frac{9!}{5! \cdot 4!}$$

Counting Rule - Combination

How many orders could 30 students sit in 9 chairs where order does not matter?

$$_NC_n = \frac{N!}{n!(N-n)!}$$

$$= \frac{30!}{9!(30-9)!} = \frac{30!}{9! \cdot 21!}$$

Assigning Probabilities to Experimental Outcomes

- ✓ Classical Method
  - Assigning probabilities based on the assumption of equally likely outcomes.
- ✓ Relative Frequency Method
  - Assigning probabilities based on **experimentation** or historical data.
- ✓ Subjective Method
  - Assigning probabilities based on the **assignor's** judgment.

Classical Method

- ✓ If an experiment has n possible outcomes, this method would assign a probability of $1/n$ to each outcome.

Example

Experiment: Rolling a die

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Probabilities: Each sample point has a 1/6 chance of occurring.

Classical Method

Rules	Example
$0 \le P(x_i) \le 1$	$P(H) \le 1$ and $P(T) \le 1$
$\sum (P(x_i)) = 1$	$P(H) + P(T) = 1$
$P(x_i) = P(x_j) = 1/N$	$P(H) = P(T)$

Homework Assignment

Read Chapter 4 Sections 1 - 4