Chapter 4

Assigning Probabilities to Experimental Outcomes

- ✓ Classical Method
  - Assigning probabilities based on the assumption of equally likely outcomes.
- ✓ Relative Frequency Method
  - Assigning probabilities based on **experimentation** or historical data.
- ✓ Subjective Method
  - Assigning probabilities based on the **assignor's** judgment.

Classical Method

- ✓ If an experiment has *n* possible outcomes, this method would assign a probability of $1/n$ to each outcome.

Example

Experiment: Rolling a die

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Probabilities: Each sample point has a 1/6 chance of occurring.

Classical Method

Rules

$0 \le P(x_i) \le 1$

$\sum (P(x_i)) = 1$

$P(x_i) = P(x_j) = 1/N$

Example

$P(H) \le 1$ and $P(T) \le 1$

$P(H) + P(T) = 1$

$P(H) = P(T)$

Relative Frequency Method

Rules

$0 \le P(x_i) \le 1$

$\sum (P(x_i)) = 1$

Example

$P(M) \le 1$ and $P(F) \le 1$

$P(M) + P(F) = 1$

$$P(x_i) = \frac{m}{n} = \frac{\text{\# of outcomes where the event occured}}{\text{\# of outcomes observed}}$$

Example: Shipwreck Subdivision

✓ Relative Frequency Method

- $P(X=x_i) = rf_i = f_i/n$
- The Shipwreck Subdivision has 40 homes.

Number of Bedrooms	Number of Houses
1	4
2	12
3	12
4	8
5	4
	40

Example: Highlands Subdivision

- ✓ Relative Frequency Method
  - $P(X=x_i) = rf_i = f_i/n$
  - The Highlands Subdivision has 40 homes.

Number of Bedrooms	Number of Houses	Notation
1	4	P(x=1)
2	12	P(x=2)
3	12	P(x=3)
4	8	P(x=4)
5	4	P(x=5)
	40	

Example: Highlands Subdivision

- ✓ Relative Frequency Method
  - $P(X=x_i) = rf_i = f_i/n$
  - The Highlands Subdivision has 40 homes.

Number of Bedrooms	Number of Houses	Notation	Probability
1	4	P(x=1)	4/40 = 0.1
2	12	P(x=2)	12/40 = 0.3
3	12	P(x=3)	12/40 = 0.3
4	8	P(x=4)	8/40 = 0.2
5	4	P(x=5)	4/40 = 0.1
	40		1.0

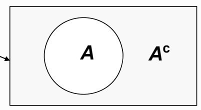
Some Basic Relationships of Probability

- ✓ Complement of an Event
- ✓ Union of Two Events
- ✓ Intersection of Two Events
- ✓ Mutually Exclusive Events

Complement of an Event

- ✓ The complement of event A is defined to be the event consisting of all sample points that are _______________.
  - The complement of A is denoted by A^c.
  - The Venn diagram below illustrates the concept of a complement.

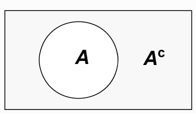
Sample Space S



Complementary Rule

$P(A^c) = 1- P(A)$

P(___________) = 1 - P(____________)



Complement of an Event

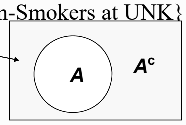
A = {Males at UNK}

A^c = {Females as UNK}

B = {all Smokers at UNK}

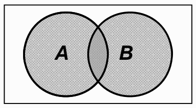
B^c = {all non-Smokers at UNK}

Sample Space S



Union of Two Events

- ✓ The union of events ***A*** and ***B*** is the event containing all sample points that are in *A* or *B* or both.
- ✓ The union is denoted by $\boldsymbol{A \cup B}$.
- ✓ The union of ***A*** and ***B*** is illustrated below.



Union of Two Events

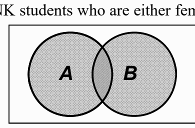
✓ Example:

$\boldsymbol{A \cup B}$ = {UNK students who are either male or who smoke}

$\boldsymbol{A^c \cup B}$ = {UNK students who are either female or who smoke}

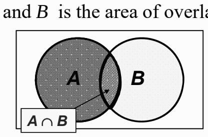
$\boldsymbol{A \cup B^c}$ = {UNK students who are either male or who do not smoke}

$\boldsymbol{A^c \cup B^c}$ = {UNK students who are either female or who do not smoke}

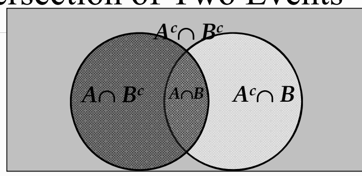


Intersection of Two Events

- ✓ The **intersection** of events *A* and *B* is the set of all sample points that are in both *A* and *B*.
- ✓ The intersection is denoted by $A \cap B$.
- ✓ The intersection of *A* and *B* is the area of overlap in the illustration below.



Intersection of Two Events



$A \cap B$ = {UNK students who are male smokers}
$A^c \cap B$ ={UNK students who are female smokers}
$A \cap B^c$ = {UNK students who are male non-smokers}
$A^c \cap B^c$ = {UNK students who are female non-smokers}

Addition Law

- ✓ The <u>addition law</u> provides a way to compute the probability of event A or B or both A and B occurring.
- ✓ The law is written as:

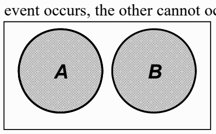
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- ✓ Example: UNK Gender and Smoking Habits

  Let $P(A) = .50$, $P(B) = .30$, $P(A \cap B) = .13$

Addition Law for Mutually Exclusive Events

- ✓ Two events are said to be <u>mutually exclusive</u> if the events have no sample points in common. That is, two events are mutually exclusive if, when one event occurs, the other cannot occur.

x


- ✓ Addition Law for Mutually Exclusive Events:

  $P(A \cup B) = P(A) + P(B)$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of A given B is denoted by $P(A|B)$.
- Example: UNK Gender and Smoking Habits
  - What is the probability of selecting a Male given that they smoke?

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} =$$

Multiplication Law

- ✓ The multiplication law provides a way to compute the probability of an intersection of two events.
- ✓ The law is written as: $P(A \cap B) = P(B)\,P(A|B)$
- Example: UNK Gender and Smoking Habits

Multiplication Law for Independent Events

- ✓ Events A and B are independent if $P(A|B) = P(A)$.
- ✓ Multiplication Law for Independent Events:

$$P(A \cap B) = P(A)P(B)$$

- ✓ The multiplication law also can be used as a test to see if two events are independent.
- ✓ Example: Are A and B independent?

Does $P(A \cap B) = P(A)P(B)$?

We know: $P(A \cap B) = .13$, $P(A) = .50$, $P(B) = .30$

But: $P(A)P(B) = (.50)(.30) = .15$

$.13 \neq .15$, so A and B are ________________________

Examples

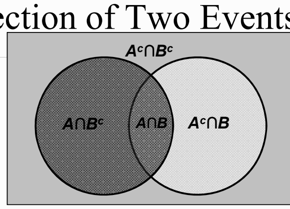
$P(A)$ = P(Condom success) $P(A)$ = ____

$P(A^c)$ = P(Condom failure) $P(A^c)$ = ____

$P(B)$ = P(Female is fertile) $P(B)$ = ____

$P(B^c)$= P(Female is not fertile) $P(B^c)$ = ____

Intersection of Two Events



$A \cap B$ = {Condoms succeed and the woman is fertile}

$A \cap B^c$ = {Condoms succeed and the woman is not fertile}

$A^c \cap B^c$ = {Condoms fail and the woman is not fertile}

$A^c \cap B$ = {Condoms fail and the woman is fertile}