Chapter 5 – Poisson Distribution

© 1996 Randy Glasbergen. E-mail: randyg@norwich.net

"Looks like we're the victims of corporate sabotage. Someone has been making decaf!"

Using the Complementary Rule

n	x	P .30
10	0	.0282
	1	.1211
	2	.2335
	3	.2668
	4	.2001
	5	.1029
	6	.0368
	7	.0090
	8	.0014
	9	.0001
	10	.0000

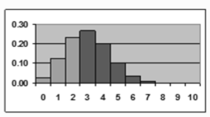
$f(x \geq 3 | n = 10, p = .30)$

$= 1 - f(x < 3 | n = 10, p = .30)$

$= 1 - f(x \leq 2 | n = 10, p = .30)$

$= 1 - (.0282 + .1211 + .2335)$

$= 1 - .3828 = .6172$



The Poisson Probability Distribution

Properties of a Poisson Experiment

- The probability of an occurrence is the same for any two ___________________of equal length.
- The occurrence or nonoccurrence in any ____________________is independent of the occurrence or nonoccurrence in any other _____________________.

The Poisson Probability Distribution

Poisson Probability Function

$$f(x \mid \mu) = \frac{\mu^x e^{-\mu}}{x!}$$

where

x = number of occurrences in an interval

μ = mean number of occurrences in an interval

e = 2.71828

Good Samaritan Hospital

Patients arrive at the emergency room of Good Samaritan Hospital at the average rate of 6 per hour on weekend evenings. What is the probability of 4 arrivals in 30 minutes on a weekend evening?

✓Using the Poisson Probability Function

μ = 6/hour = 3/half-hour, x = 4

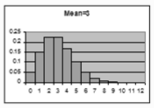
$$f(x = 4 \text{ Arrivals per ½ hour} \mid \mu = 3 \text{ Arrivals per ½ hour}) = \frac{3^4 e^{-3}}{4!} =$$

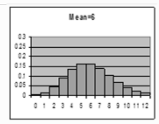
Using the Tables of Poisson Probabilities

	μ									
x	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1494
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	**.1680**
5	.0417	.0476	.0538	.0602	..0668	.0735	.0804	.0872	.0940	.1008
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

Using the Tables of Poisson Probabilities

x	μ = 3.0	μ = 6.0
0	.0498	.0025
1	.1494	.0149
2	.2240	.0446
3	.2240	.0892
4	.1680	.1339
5	.1008	.1606
6	.0504	.1606
7	.0216	.1377
8	.0081	.1033
9	.0027	.0688
10	.0008	.0413
11	.0002	.0225
12	.0001	.0113





Good Samaritan Hospital

What is the probability of 7 arrivals in one hour on a weekend evening?

μ = ________, x = _________

Using the Tables of Poisson Probabilities

x	μ = 5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027	.0025
1	.0311	.0287	.0265	.0244	.0225	.0207	.0191	.0176	.0162	.0149
2	.0793	.0746	.0701	.0659	.0618	.0580	.0544	.0509	.0477	.0446
3	.1348	.1293	.1239	.1185	.1133	.1082	.1033	.0985	.0938	.0892
4	.1719	.1681	.1641	.1600	.1558	.1515	.1472	.1428	.1383	.1339
5	.1753	.1748	.1740	.1728	.1714	.1697	.1678	.1656	.1632	.1606
6	.1490	.1515	.1537	.1555	.1571	.1584	.1594	.1601	.1605	.1606
7	.1086	.1125	.1163	.1200	.1234	.1267	.1298	.1326	.1353	**.1377**
8	.0692	.0731	.0771	.0810	.0849	.0887	.0925	.0962	.0998	.1033
9	.0392	.0423	.0454	.0486	.0519	.0552	.0586	.0620	.0654	.0688
10	.0200	.0220	.0241	.0262	.0285	.0309	.0334	.0359	.0386	.0413
11	.0093	.0104	.0116	.0129	.0143	.0157	.0173	.0190	.0207	.0225
12	.0039	.0045	.0051	.0058	.0065	.0073	.0082	.0092	.0102	.0113

Using Excel

What is the probability of 7 or fewer arrivals in one hour on a weekend evening?

What is the probability of 8 or more arrivals in one hour on a weekend evening?

Complementary Rule

$$f(x \geq 8 \mid \mu = 6) = 1 - f(x \leq 7 \mid \mu = 6)$$

Using the Tables of Poisson Probabilities

	μ									
x	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027	.0025
1	.0311	.0287	.0265	.0244	.0225	.0207	.0191	.0176	.0162	.0149
2	.0793	.0746	.0701	.0659	.0618	.0580	.0544	.0509	.0477	.0446
3	.1348	.1293	.1239	.1185	.1133	.1082	.1033	.0985	.0938	.0892
4	.1719	.1681	.1641	.1600	.1558	.1515	.1472	.1428	.1383	.1339
5	.1753	.1748	.1740	.1728	.1714	.1697	.1678	.1656	.1632	.1606
6	.1490	.1515	.1537	.1555	.1571	.1584	.1594	.1601	.1605	.1606
7	.1086	.1125	.1163	.1200	.1234	.1267	.1298	.1326	.1353	**.1377**
8	.0692	.0731	.0771	.0810	.0849	.0887	.0925	.0962	.0998	.1033
9	.0392	.0423	.0454	.0486	.0519	.0552	.0586	.0620	.0654	.0688
10	.0200	.0220	.0241	.0262	.0285	.0309	.0334	.0359	.0386	.0413
11	.0093	.0104	.0116	.0129	.0143	.0157	.0173	.0190	.0207	.0225
12	.0039	.0045	.0051	.0058	.0065	.0073	.0082	.0092	.0102	.0113

The Poisson Probability Distribution

- ✓ Expected Value

  $E(x) = \mu$

- ✓ Variance

  $\text{Var}(x) = \sigma^2 = \mu$

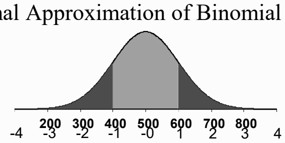
- ✓ Standard Deviation

  $\text{SD}(x) = \sigma = \sqrt{\mu}$

Chapter 6
Continuous Probability Distributions

Continuous Probability Distributions

- ✓ The Uniform Probability Distribution
- ✓ The Normal Probability Distribution
- ✓ Normal Approximation of Binomial Probabilities



Continuous Probability Distributions

- ✓ A ________________ random variable can assume any value in an interval on the real line or in a collection of intervals.
- ✓ It is not possible to talk about the probability of the random variable assuming a particular value.
- ✓ Instead, we talk about the probability of the random variable assuming a value within a given interval.
- ✓ The probability of the random variable assuming a value within some given interval from *a* to *b* is defined to be the area under the graph of the probability density function between *a* and *b*.

Uniform Probability Distribution

A random variable is <u>uniformly distributed</u> whenever the probability is proportional to the length of the interval.

Uniform Probability Distribution

- ✓ Uniform Probability Density Function

  $f(x) = 1/(b - a)$ for $a \leq x \leq b$

  $= 0$ elsewhere

- ✓ Expected Value of x

  $E(x) = (a + b)/2$

- ✓ Variance of x

  $Var(x) = (b - a)^2/12$

  where

  a = smallest value the variable can assume

  b = largest value the variable can assume

Valentinos Salad Buffet

Valentinos customers are charged for the amount of salad they take. Sampling suggests that the amount of salad taken is uniformly distributed between 5 ounces and 15 ounces.

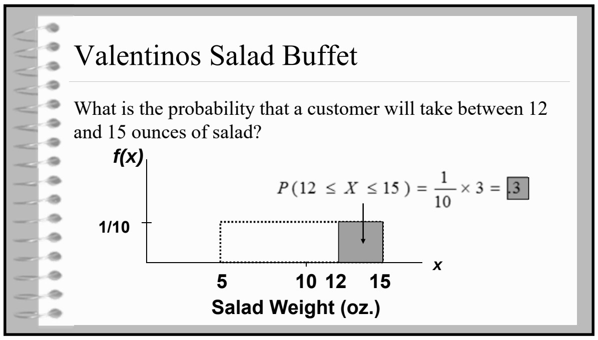
- ✓ Probability Density Function

  $f(x) = 1/10$ for $5 \leq x \leq 15$

  $= 0$ elsewhere

  where

  x = salad plate filling weight


Valentinos Salad Buffet
What is the probability that a customer will take between 12 and 15 ounces of salad?
f(x)
$P(12 \le X \le 15) = \frac{1}{10} \times 3 = .3$
1/10
x
5
10 12
15
Salad Weight (oz.)