Chapter 7

Abraham Lincoln:
If this is coffee, please bring me some tea; but if this is tea, please bring me some coffee.

Anne Morrow Lindbergh
Good communication is as stimulating as black coffee, and just as hard to sleep after.

T. S. Eliot (1888-1965)
I have measured out my life with coffee spoons.

Chapter 7
Sampling and Sampling Distributions

- ✓ Simple Random Sampling
- ✓ Point Estimation
- ✓ Introduction to Sampling Distributions
- ✓ Sampling Distribution of $\bar{x}$
- ✓ Sampling Distribution of ρ
- ✓ Properties of Point Estimators
- ✓ Other Sampling Methods

Statistical Inference

- ✓ The purpose of ____________________ is to obtain information about a population from information contained in a sample.
- ✓ The sample results provide only ______________ of the values of the population characteristics.
- ✓ A <u>parameter</u> is a numerical characteristic of a population.
- ✓ With <u>proper sampling methods</u>, the sample results will provide “good” estimates of the population characteristics.

Simple Random Sampling

✓ Finite Population

- A simple random sample from a finite population of size N is a sample selected such that each possible sample of size n has the same probability of being selected.
- sampling with replacement - Replacing each sampled element before selecting subsequent elements
- Sampling without replacement is the procedure used most often.

Simple Random Sampling

✓ Infinite Population

- A simple random sample from an ___________population is a sample selected such that the following conditions are satisfied.
  - Each element selected comes from the same population.
  - Each element is selected independently.

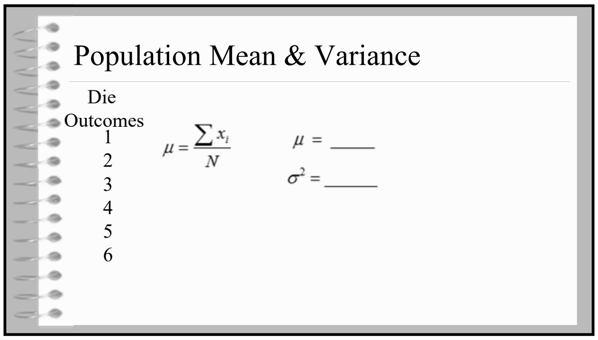
Point Estimation

In point estimation, a sample statistic that serves as an estimate of a population parameter.

$\bar{x}$ is the point estimator of μ

s is the point estimator of σ

$\bar{p}$ is the point estimator of p



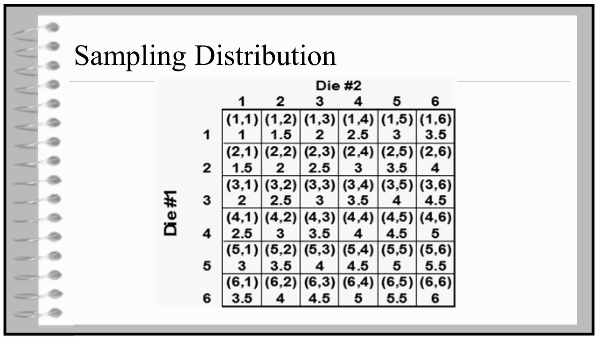
Population Mean & Variance

Die
Outcomes
1
2
3
4
5
6

$\mu = \frac{\sum x_i}{N}$

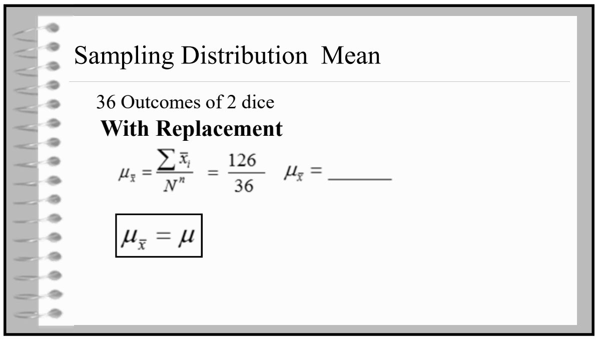
$\mu =$ _____

$\sigma^2 =$ ______



Sampling Distribution

				Die #2			
		1	2	3	4	5	6
Die #1	1	(1,1) 1	(1,2) 1.5	(1,3) 2	(1,4) 2.5	(1,5) 3	(1,6) 3.5
	2	(2,1) 1.5	(2,2) 2	(2,3) 2.5	(2,4) 3	(2,5) 3.5	(2,6) 4
	3	(3,1) 2	(3,2) 2.5	(3,3) 3	(3,4) 3.5	(3,5) 4	(3,6) 4.5
	4	(4,1) 2.5	(4,2) 3	(4,3) 3.5	(4,4) 4	(4,5) 4.5	(4,6) 5
	5	(5,1) 3	(5,2) 3.5	(5,3) 4	(5,4) 4.5	(5,5) 5	(5,6) 5.5
	6	(6,1) 3.5	(6,2) 4	(6,3) 4.5	(6,4) 5	(6,5) 5.5	(6,6) 6



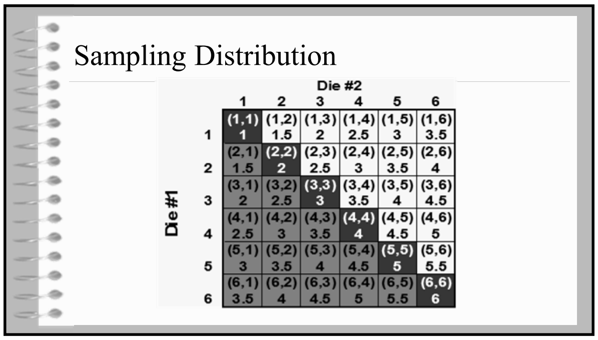
Sampling Distribution Mean

36 Outcomes of 2 dice

With Replacement

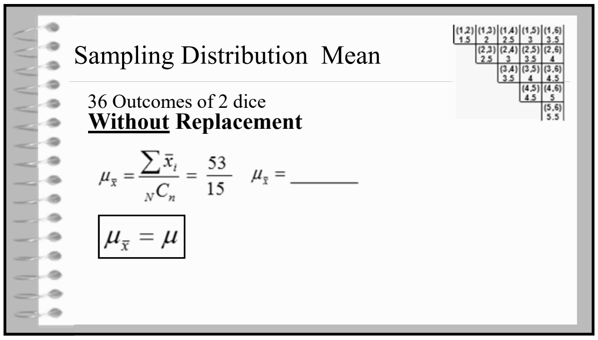
$\mu_{\bar{x}} = \frac{\sum \bar{x}_i}{N^n} = \frac{126}{36}$ $\mu_{\bar{x}} =$ _______

$$\boxed{\mu_{\bar{x}} = \mu}$$



Sampling Distribution

Die #1	Die #2 1	2	3	4	5	6
1	(1,1) 1	(1,2) 1.5	(1,3) 2	(1,4) 2.5	(1,5) 3	(1,6) 3.5
2	(2,1) 1.5	(2,2) 2	(2,3) 2.5	(2,4) 3	(2,5) 3.5	(2,6) 4
3	(3,1) 2	(3,2) 2.5	(3,3) 3	(3,4) 3.5	(3,5) 4	(3,6) 4.5
4	(4,1) 2.5	(4,2) 3	(4,3) 3.5	(4,4) 4	(4,5) 4.5	(4,6) 5
5	(5,1) 3	(5,2) 3.5	(5,3) 4	(5,4) 4.5	(5,5) 5	(5,6) 5.5
6	(6,1) 3.5	(6,2) 4	(6,3) 4.5	(6,4) 5	(6,5) 5.5	(6,6) 6



Sampling Distribution Mean

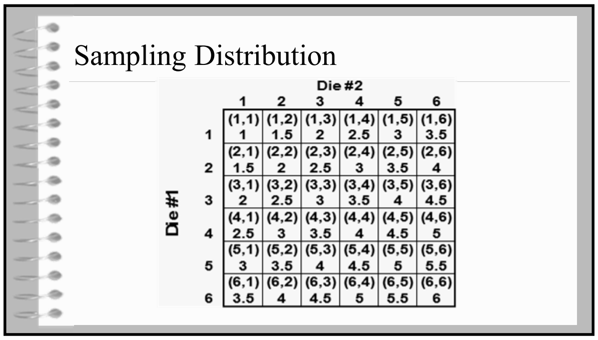
(1,2) 1.5	(1,3) 2	(1,4) 2.5	(1,5) 3	(1,6) 3.5
	(2,3) 2.5	(2,4) 3	(2,5) 3.5	(2,6) 4
		(3,4) 3.5	(3,5) 4	(3,6) 4.5
			(4,5) 4.5	(4,6) 5
				(5,6) 5.5

36 Outcomes of 2 dice

<u>Without</u> Replacement

$$\mu_{\bar{x}} = \frac{\sum \bar{x}_i}{{}_N C_n} = \frac{53}{15} \qquad \mu_{\bar{x}} = \_\_\_\_\_\_\_$$

$$\boxed{\mu_{\bar{x}} = \mu}$$



Sampling Distribution

Die #1	Die #2 1	2	3	4	5	6
1	(1,1) 1	(1,2) 1.5	(1,3) 2	(1,4) 2.5	(1,5) 3	(1,6) 3.5
2	(2,1) 1.5	(2,2) 2	(2,3) 2.5	(2,4) 3	(2,5) 3.5	(2,6) 4
3	(3,1) 2	(3,2) 2.5	(3,3) 3	(3,4) 3.5	(3,5) 4	(3,6) 4.5
4	(4,1) 2.5	(4,2) 3	(4,3) 3.5	(4,4) 4	(4,5) 4.5	(4,6) 5
5	(5,1) 3	(5,2) 3.5	(5,3) 4	(5,4) 4.5	(5,5) 5	(5,6) 5.5
6	(6,1) 3.5	(6,2) 4	(6,3) 4.5	(6,4) 5	(6,5) 5.5	(6,6) 6

Sampling Distribution Variance

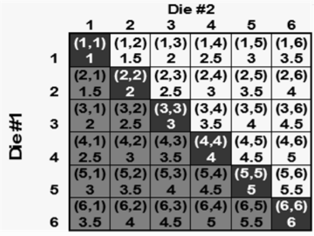
36 Outcomes of 2 dice

With Replacement

$$\sigma_{\bar{x}}^2 = \frac{\sum(\bar{x}_i - \mu_{\bar{x}})^2}{N^n} \qquad = \frac{525}{36} \qquad \sigma_{\bar{x}}^2 = 1.458\overline{3}$$

$$\boxed{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}} \qquad \sigma_{\bar{x}}^2 = \frac{2.91\overline{6}}{2} = 1.458\overline{3}$$

Sampling Distribution



Die #1 \ Die #2	1	2	3	4	5	6
1	(1,1) 1	(1,2) 1.5	(1,3) 2	(1,4) 2.5	(1,5) 3	(1,6) 3.5
2	(2,1) 1.5	(2,2) 2	(2,3) 2.5	(2,4) 3	(2,5) 3.5	(2,6) 4
3	(3,1) 2	(3,2) 2.5	(3,3) 3	(3,4) 3.5	(3,5) 4	(3,6) 4.5
4	(4,1) 2.5	(4,2) 3	(4,3) 3.5	(4,4) 4	(4,5) 4.5	(4,6) 5
5	(5,1) 3	(5,2) 3.5	(5,3) 4	(5,4) 4.5	(5,5) 5	(5,6) 5.5
6	(6,1) 3.5	(6,2) 4	(6,3) 4.5	(6,4) 5	(6,5) 5.5	(6,6) 6

Sampling Distribution Variance

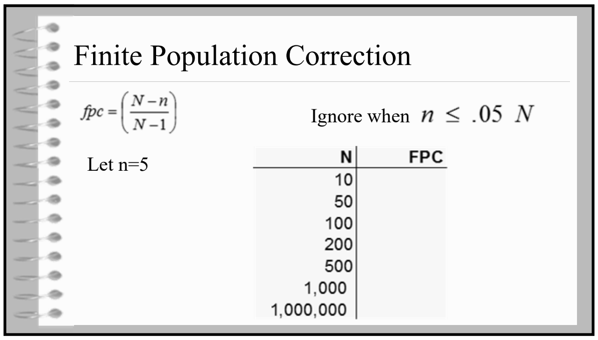
(1,2) 1.5	(1,3) 2	(1,4) 2.5	(1,5) 3	(1,6) 3.5
	(2,3) 2.5	(2,4) 3	(2,5) 3.5	(2,6) 4
		(3,4) 3.5	(3,5) 4	(3,6) 4.5
			(4,5) 4.5	(4,6) 5
				(5,6) 5.5

Outcomes of 2 dice

Without Replacement

$$\sigma_{\bar{x}}^2 = \frac{\sum(\bar{x}_i - \mu_{\bar{x}})^2}{{}_NC_n} = \frac{17.5}{15} \qquad \sigma_{\bar{x}}^2 = 1.1\overline{6}$$

$$\boxed{\sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n}} \qquad \sigma_{\bar{x}}^2 = \left(\frac{4}{5}\right)\frac{2.91\overline{6}}{2} = 1.1\overline{6}$$



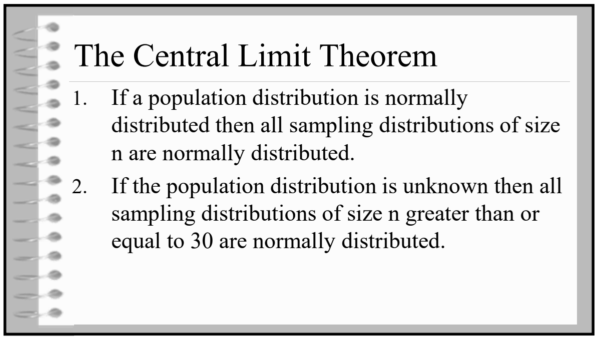
Finite Population Correction

$fpc = \left(\frac{N-n}{N-1} \right)$

Ignore when $n \leq .05\ N$

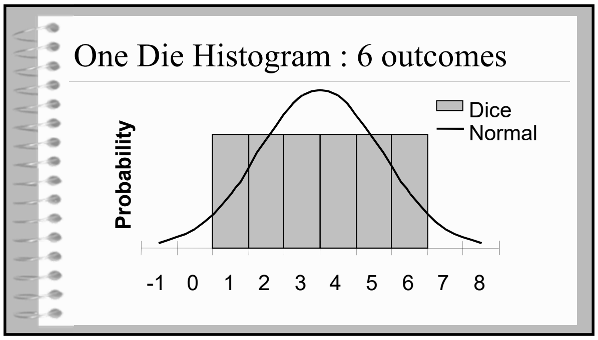
Let n=5

N	FPC
10	
50	
100	
200	
500	
1,000	
1,000,000	

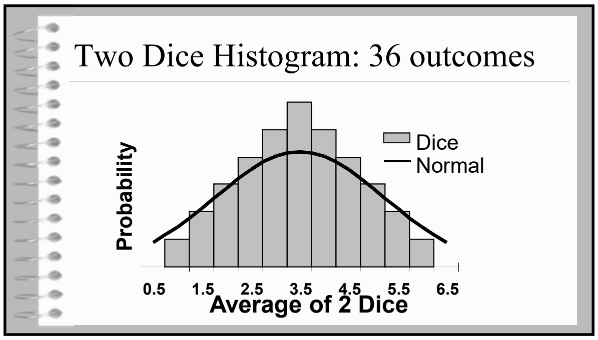


The Central Limit Theorem

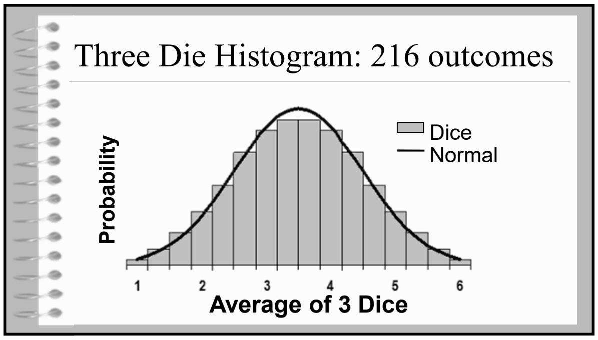
1. If a population distribution is normally distributed then all sampling distributions of size n are normally distributed.
2. If the population distribution is unknown then all sampling distributions of size n greater than or equal to 30 are normally distributed.



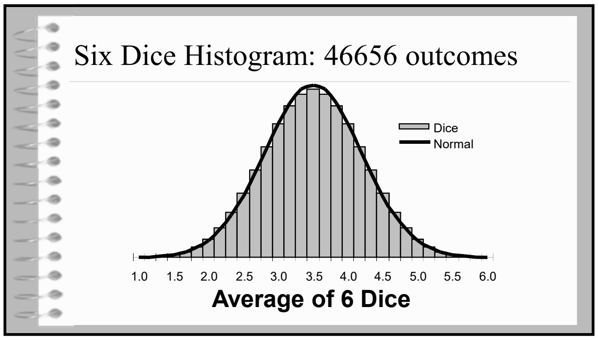
One Die Histogram : 6 outcomes



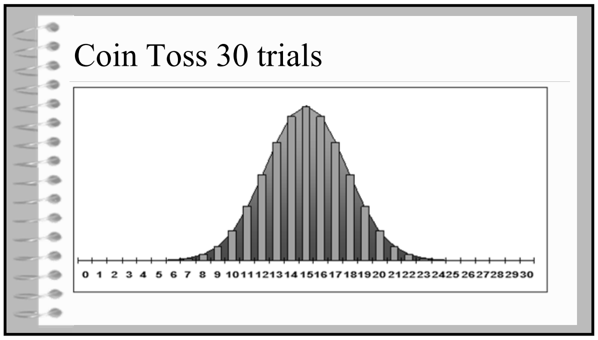
Two Dice Histogram: 36 outcomes



Three Die Histogram: 216 outcomes



Six Dice Histogram: 46656 outcomes



Coin Toss 30 trials