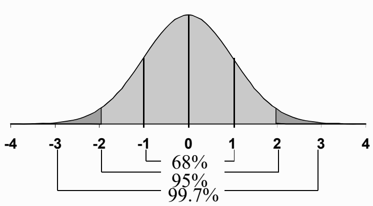
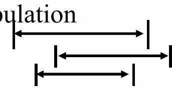
Chapter 8 Interval Estimation



Interval Estimation - Outline

- ✓ Interval Estimation of a Population Mean:
  - Large-Sample Case
  - Small-Sample Case
- ✓ Determining the Sample Size
- ✓ Interval Estimation of a Population Proportion



Interval Estimation of a Population Mean: Large-Sample Case

- ✓ Sampling Error
- ✓ Probability Statements about the Sampling Error
- ✓ Calculating an Interval Estimate:
  - Large-Sample Case with σ Known
  - Large-Sample Case with σ Unknown

Sampling Error

Sampling Error = $|\bar{x} - \mu|$

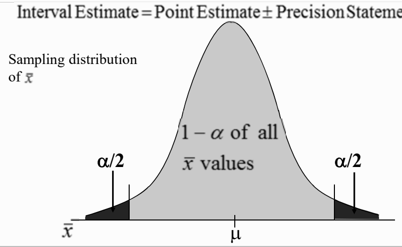
- ✓ The absolute value of the difference between an unbiased point estimate and the population parameter it estimates is called the sampling error.

Probability Statements About the Sampling Error

- ✓ Knowledge of the sampling distribution of $\bar{x}$ enables us to make probability statements about the sampling error even though the population mean μ is not known.
- ✓ A probability statement about the sampling error is a precision statement.

Probability Statements About the Sampling Error

Interval Estimate = Point Estimate ± Precision Statement



Example: The Buckle, Inc.

The Buckle, inc. has about 500 retail outlets throughout the United States. The corporate officers evaluates each potential location for a new retail outlet in part on the mean annual income of the individuals in the marketing area of the new location.

The purpose of this example is to show how sampling can be used to develop an interval estimate of the mean annual income for individuals in a potential marketing area for The Buckle.

Based on similar annual income surveys, the standard deviation of annual incomes in the entire population is considered known with σ = $5,000.

We will use a sample size of n = 64.

The Buckle, Inc

✓ Precision Statement

There is a .95 probability that the value of a sample mean for The Buckle will provide a sampling error of $1,225 or less.

Determined as follows:

95% of the sample means that can be observed are within $\pm$ 1.96$\sigma_{\bar{x}}$ of the population mean μ.

If $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 5{,}000/\sqrt{64} = 625$

then 1.96 $\sigma_{\bar{x}}$ =

Interval Estimate of a Population Mean: Large-Sample Case ($n \geq 30$)

- With σ Known $\quad \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

  where: $1 - \alpha$ is the confidence coefficient

  $z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal probability distribution

- With σ Unknown $\quad \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$

  where: s is the sample standard deviation

Example: The Buckle, Inc.

- Interval Estimate of the Population Mean: σ Known

Assume that the sample mean, $\bar{x}$, is \$21,100. Recall that the sampling error value, $1.96\sigma_{\bar{x}}$, in our precision statement is \$1,225.

Interval Estimate of μ is \$21,100 ± \$1,225

$P(\$\_\_\_\_\_\_\_\_\_ \leq \mu \leq \$\_\_\_\_\_\_\_\_\_) = .95$

We are <u>95% confident</u> that the average annual income of the individuals in the marketing area of the new location is between \$_________ and \$_________.

The Buckle, Inc

- Interval Estimate – unknown population variance

Determine a 90% interval estimate for the mean annual income of the individuals in the marketing area of the new location. A survey of size 64 is performed resulting in a sample mean income of \$21,100 and a sample standard deviation of \$5000.

$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} \qquad \bar{x} \pm z_{\alpha/2}s_{\bar{x}} \qquad s_{\bar{x}} = \frac{s}{\sqrt{n}} = \_\_\_\_\_\_ =$

$z_{\alpha/2} =$

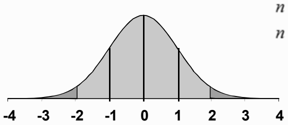
Comparison of *n* and α

$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$ as $\alpha\uparrow,\ z\downarrow$

$\alpha\downarrow,\ z\uparrow$

$n\uparrow,\ s_{\bar{x}}\downarrow$

$n\downarrow,\ s_{\bar{x}}\uparrow$



Interval Estimation of a Population Mean: Small-Sample Case ($n < 30$)

- ✓ If Population is not normally distributed:
  - The only option is to increase the sample size to $n \geq 30$ and use the large-sample interval-estimation procedures.
- ✓ If Population is normally distributed and σ is known:
  - The large-sample interval-estimation procedure can be used.
- ✓ If Population is normally distributed and σ is unknown:
  - The appropriate interval estimate is based on a probability distribution known as the *t* distribution.