Chapter 8 Interval Estimation

The t-distribution is a family of similar probability distributions.
A specific t-distribution depends on a parameter known as the degrees of freedom.
As the number of degrees of freedom increases, the difference between the t-distribution and the standard normal probability distribution becomes smaller and smaller.
A t-distribution with more degrees of freedom has less dispersion.
The mean of the t-distribution is zero.

Interval Estimation of a Population Mean: Small-Sample Case (n < 30) with \( \sigma \) Unknown

The interval estimate is given by:

\[
\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}
\]

Where 1 - \( \alpha \) is the confidence coefficient
\( \alpha/2 \) is the t value providing an area of \( \alpha/2 \) in the upper tail of a t-distribution with \( n - 1 \) degrees of freedom
\( s \) is the sample standard deviation
Example: Apartment Rents

A reporter for a student newspaper is writing an article on the cost of off-campus housing. A sample of 10 one-bedroom units within a half-mile of campus resulted in a sample mean of $350 per month and a sample standard deviation of $30.

Let us provide a 95% confidence interval estimate of the mean rent per month for the population of one-bedroom units within a half-mile of campus. We'll assume this population is normally distributed.

\[
\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}
\]

\[
\text{At 95% confidence, } 1 - \alpha = .95, \alpha = .05, \text{ and } \alpha/2 = .025.
\]

\[
t_{.025} \text{ is based on } n - 1 = 10 - 1 = 9 \text{ degrees of freedom.}
\]

In the \( t \) distribution table we see that \( t_{.025} = 2.262 \).

\[
\text{We are 95% confident that the average rent per month for the population of one-bedroom units within a half-mile of campus is between $328.54 and $371.46.}
\]
Sample Size for an Interval Estimate of a Population Mean

- Let $E$ = the maximum sampling error mentioned in the precision statement.
- $E$ is the amount added to and subtracted from the point estimate to obtain an interval estimate.
- $E$ is often referred to as the margin of error.
- We have $E = z_{a/2} \frac{\sigma}{\sqrt{n}}$
- Solving for $n$ we have $n = \left(\frac{z_{a/2} \sigma}{E}\right)^2$

Using the $t$ table to find $z$ values

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>.10</th>
<th>.05</th>
<th>.025</th>
<th>.01</th>
<th>.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.281</td>
<td>1.645</td>
<td>1.960</td>
<td>2.326</td>
<td>2.576</td>
</tr>
<tr>
<td>infinity</td>
<td>1.282</td>
<td>1.645</td>
<td>1.960</td>
<td>2.326</td>
<td>2.576</td>
</tr>
</tbody>
</table>

Example: The Buckle, Inc.

Suppose that Buckle’s management team wants an estimate of the population mean income such that there is a .95 probability that the sampling error is $500 or less. How large a sample size is needed to meet the required precision?
Example: The Buckle, Inc.

Sample Size for Interval Estimate of a Population Mean

\[ n = \left( \frac{z_{\alpha/2}}{E} \right)^2 \sigma^2 \]

At 95% confidence, 
\[ z_{0.025} = 1.96, \ E = \$500, \ \sigma = 5000 \] (recall from past example)

\[ n = \left( \frac{1.96}{5.00} \right)^2 = \frac{1}{(500)^2} \]

We need to sample _________ to reach a desired precision of $500 at 95% confidence.

Example: Political Science, Inc.

Three rules for estimating a Population Standard Deviation

1. Use \( \sigma \) from another similar study.
2. Use \( s \) from a pilot study, let \( s \approx \sigma \)
3. Let \( s \approx \frac{\text{range}}{6} \)

Interval Estimation of a Population Proportion

The interval estimate is given by:

\[ \bar{p} \pm z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} \]

where \( 1-\alpha \) is the confidence coefficient
\[ z_{\alpha/2} \] is the \( z \) value providing an area of \( \alpha/2 \) in the upper tail of the standard normal probability distribution is the sample proportion
Political Science, Inc. (PSI) specializes in voter polls and surveys designed to keep political office seekers informed of their position in a race. Using telephone surveys, interviewers ask registered voters who they would vote for if the election were held that day.

In a recent election campaign, PSI found that 220 registered voters, out of 500 contacted, favored a particular candidate. PSI wants to develop a 95% confidence interval estimate for the proportion of the population of registered voters that favors the candidate.

Example: Political Science, Inc.

Interval Estimate of a Population Proportion

\[ \hat{p} = \pm z_{0.025} \sqrt{\frac{p(1-p)}{n}} \]

PSI is 95% confident that between 39.65% and 48.35% of all voters that favors the candidate.

Sample Size for an Interval Estimate
of a Population Proportion

- Let \( E \) = the maximum sampling error mentioned in the precision statement.
- We have
  \[ E = z_{0.025} \sqrt{\frac{p(1-p)}{n}} \]
- Solving for \( n \) we have
  \[ n = \left( \frac{z_{0.025}}{E} \right)^2 \frac{p(1-p)}{\hat{p}(1-\hat{p})} \]
Suppose that PSI would like a .99 probability that the sample proportion is within \( \pm .03 \) of the population proportion. A similar study performed on the same candidate in the same region resulted in a sample proportion of 44%. How large a sample size is needed to meet the required precision?

Example: Political Science, Inc.

Sample Size for Interval Estimate of a Population Proportion

1. Use \( p \) from another similar study.
2. If no estimate of \( p \) is available, set \( p \) to .50
3. Use \( \bar{p} \) from a pilot study, let \( \bar{p} \approx p \)

At 99% confidence, \( z_{.005} = 2.576 \). Note, .44 is the best estimate of \( p \) in the above expression.

If no information is available about \( p \), what would you recommended as the size of \( n \)?

Example: Political Science, Inc.

Three rules for estimating a Population Proportion

1. Use \( p \) from another similar study.
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Sample Size for Interval Estimate of a Population Proportion

At 99% confidence, \( z_{.005} = 2.576 \). Note, .44 is the best estimate of \( p \) in the above expression.

If no information is available about \( p \), what would you recommended as the size of \( n \)?

In class exercise – Kearney 6th Graders

Each member of a random sample of 25 sixth-graders in Kearney keeps a record for one week of the amount of time spent watching television. The sample mean and sample standard deviation computed from the results are 15 hours and 9 hours respectively. Construct a 95% confidence interval for the population mean.